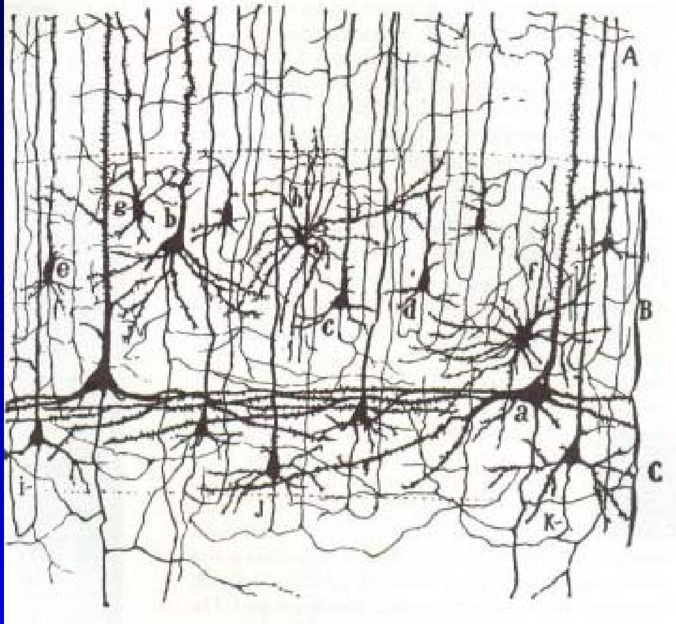
# Critical brain networks

Dante R. Chialvo

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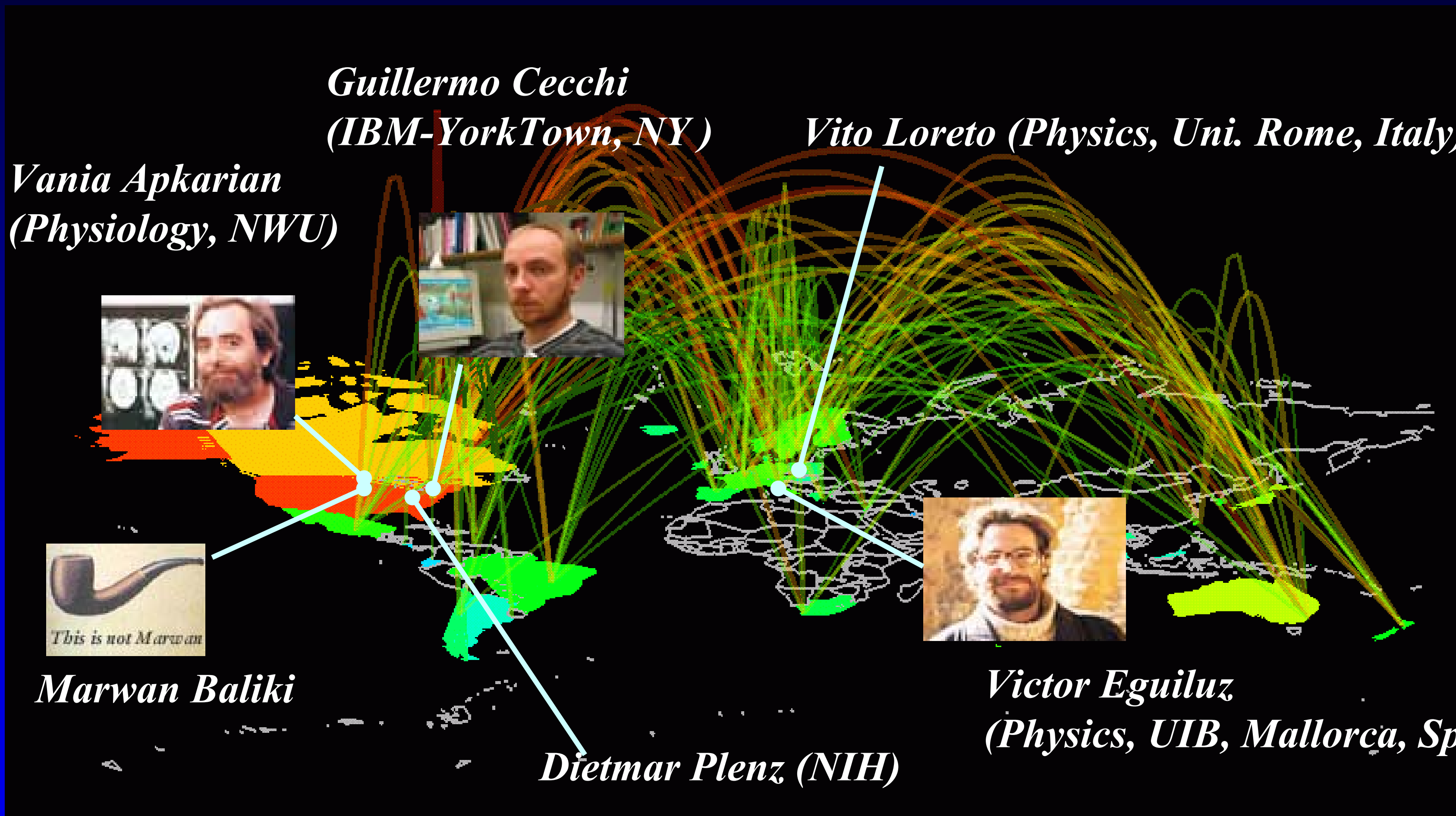
# We study networks of brain large-scale organization

How is the large scale structure of brain networks?

We examine:

- Catalogues of connectivity maps.
- Networks extracted from Functional Magnetic Resonance Imaging (Fmri).
- Neocortical cultures (with Dietmar Plenz, NIH).
- Abstract Networks.

# Credits



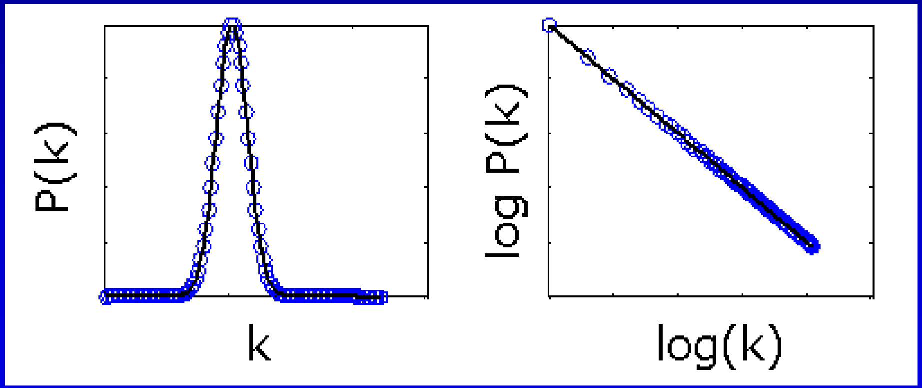


**Supported by:**
**NIH NINDS**, Endo, & Pfizer.

# *Hoja 0 del evangelio*

La estadistica que aprendimos describen **uniformidad (gaussianas, exponentiales, :"una forma")**

**Naturaleza es NO uniforme : "muchas formas"!!**
**Complejidad es no-uniformidad**
**Ejemplo: distribucion of pesos vs. distribucion de $**




**Ilustraremos esto con resultados de la corteza cerebral .**

Preguntas para despues:

*La leyes de la física son simples, como es entonces que el mundo en que estamos inmersos es complejo?*
*Como se genera complejidad a partir de leyes simples?*

# But first: Complicated or Complex?

Complicated system

many **linear** pieces + a central supervisor + blueprint = "whole"

Example: a tv set

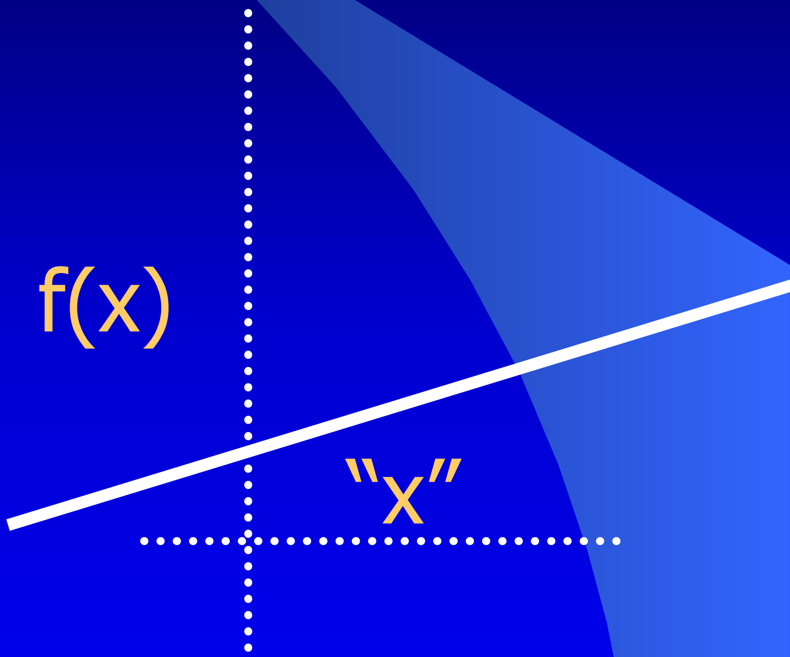
Complex system

many **nonlinear** pieces + coupling + injected energy = emergent properties

Example: society

# Lineal o No-Lineal, a quien le importa?

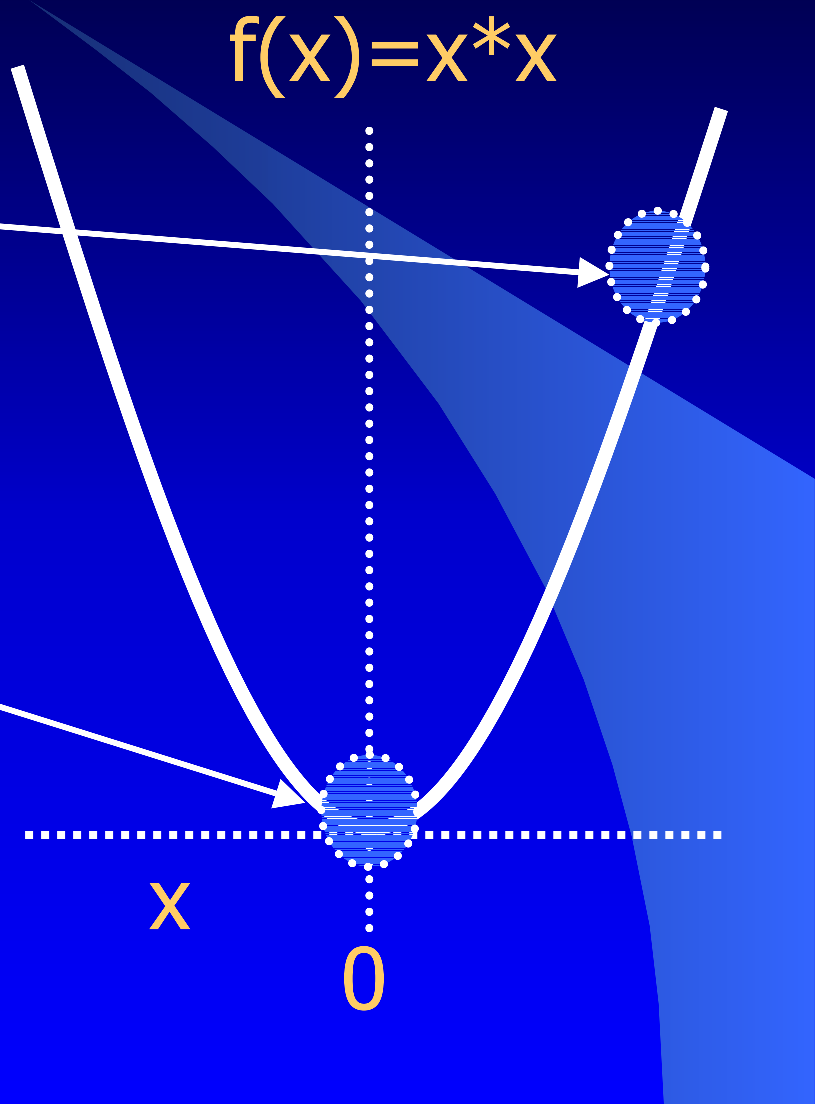
f(x) es “lineal” cuando para todo los valores de “x” la función “f” no cambia



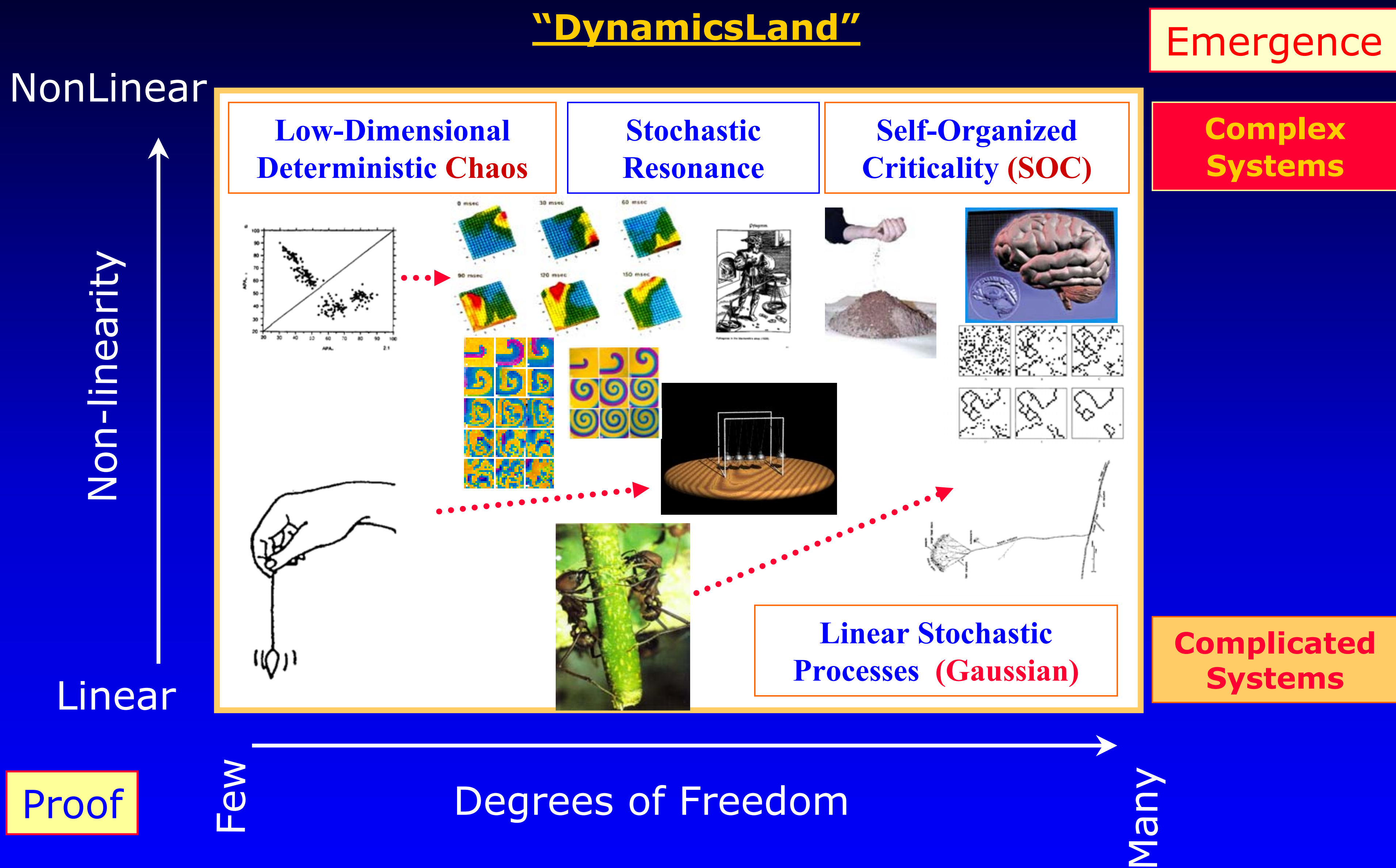

sorry

# No lineal

- para x grandes la pendiente es grande
- para x cercano a cero la pendiente es cercana a cero;



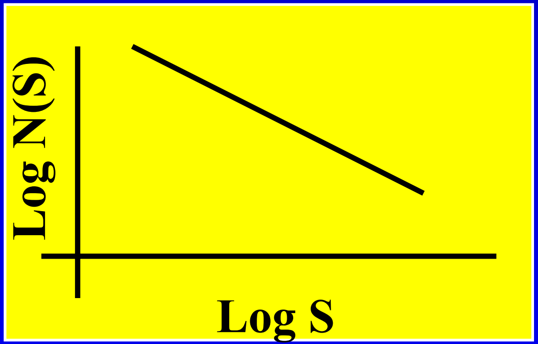
# About the idea of complex vs. complicated



# Critical (sandpile toy model*

- Drop sand slowly… nothing happen …eventually the pile will reach a state in which the addition of a single grain will produce avalanches of all sizes:

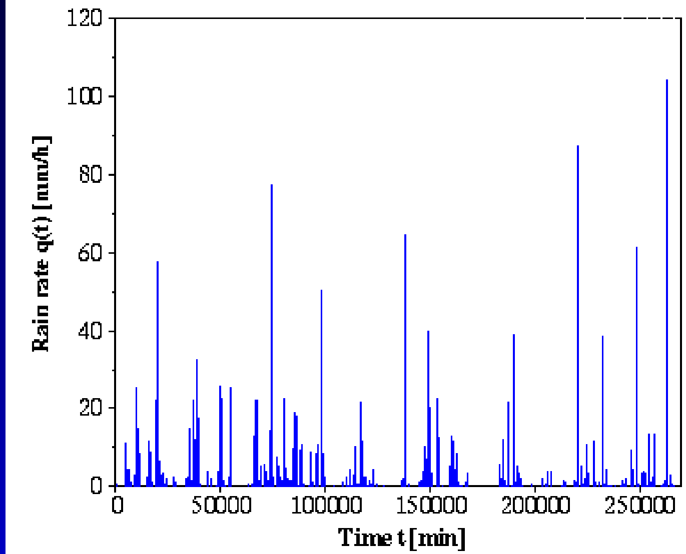
$$N(S) \approx \frac{1}{S^{\alpha}} \longrightarrow$$

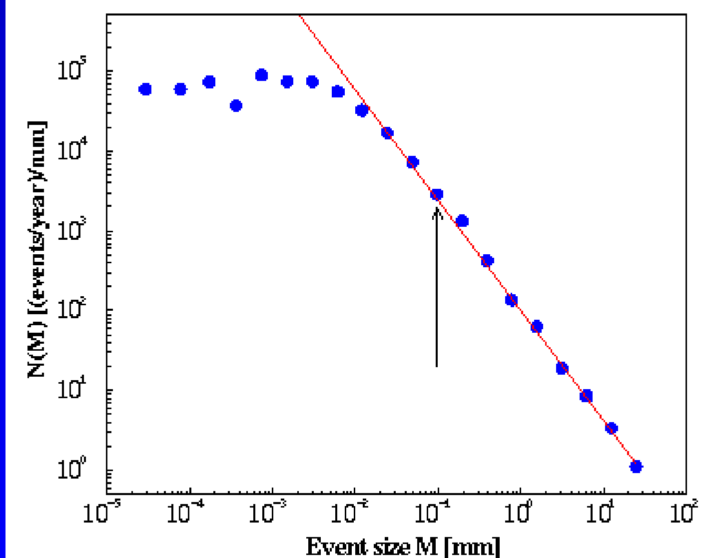



- ***N(S)*** is the number of avalanches of size ***S*** and $\alpha$ is the critical exponent.

*BTW 1987, PRL

# Another example: Rain as 'Earthquakes in the Sky'*



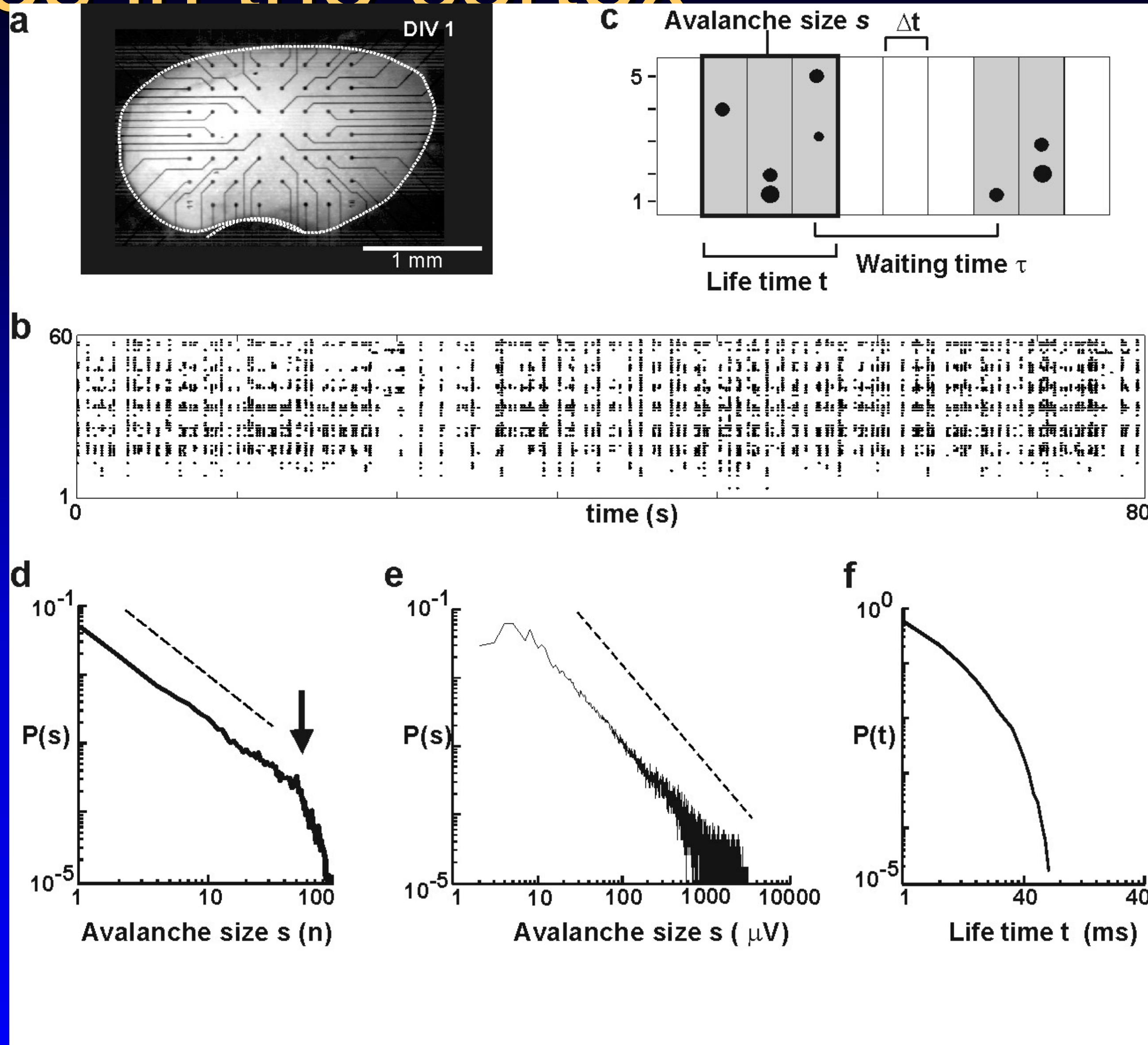




- Rain dynamics is equivalent to the Gutenberg-Richter law for earthquakes and the scale-free distribution of avalanche sizes in sandpiles

***Figures from www.cmth.ph.ic.ac.uk/kim O. Peters, C. Hertlein, and K. Christensen, *A complexity view of rainfall, Phys. Rev. Lett.* 88, 018701, 1-4 (2002).**

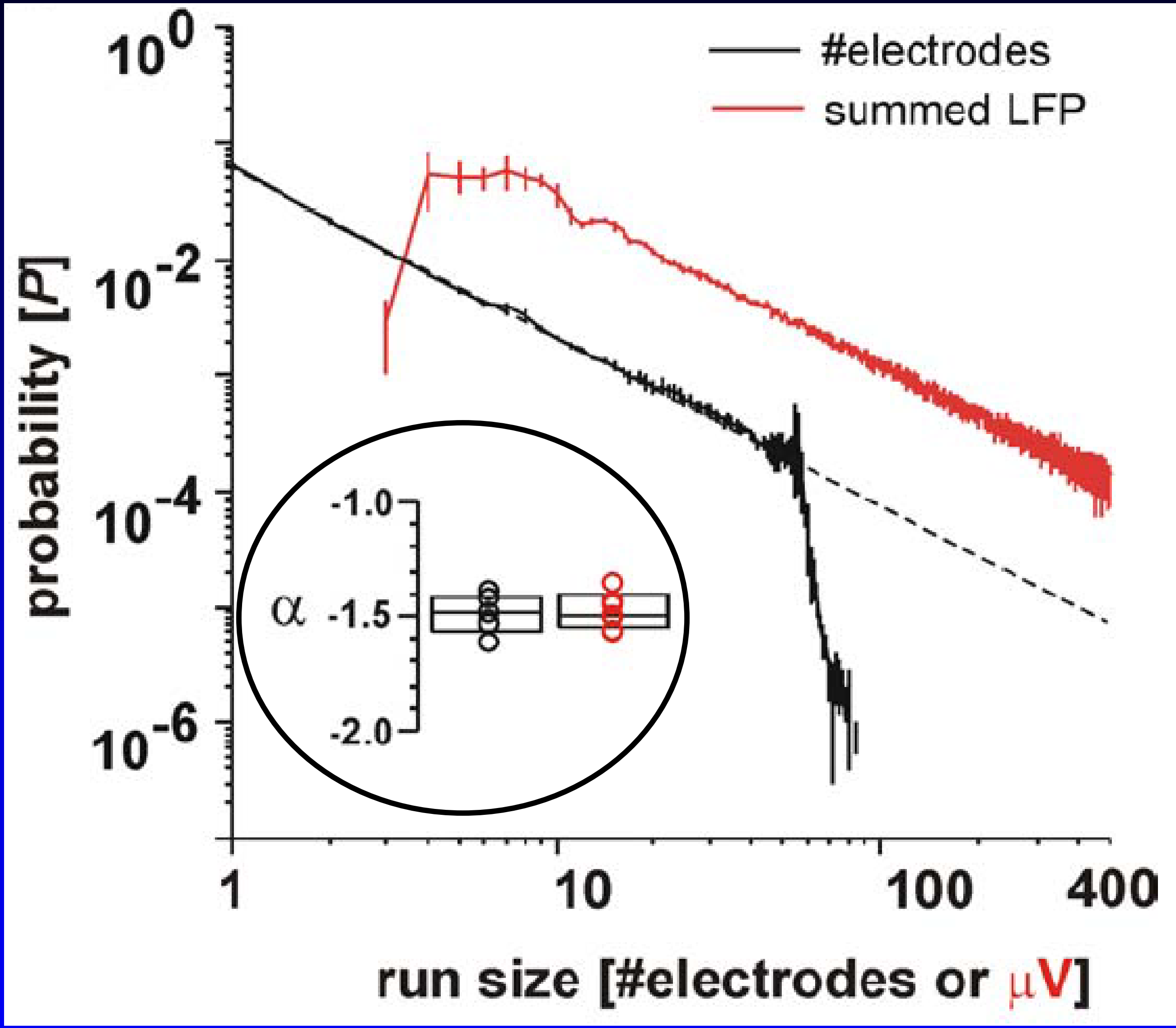
# 'Earthquakes in the cortex'

"Neuronal avalanches"




*From Plenz & Chialvo (submitted, 2004)*

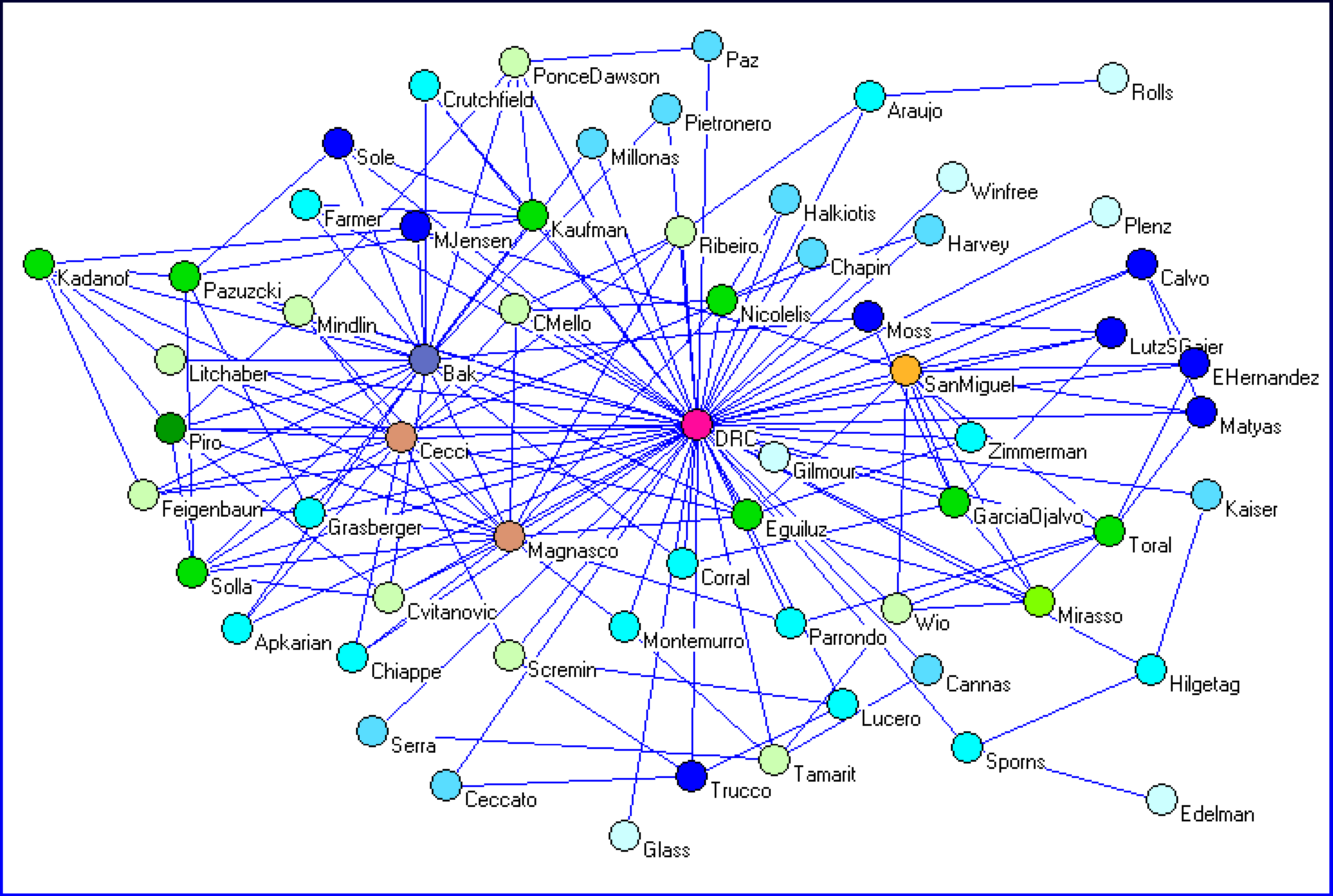
# More Earthquakes in the cortex‘



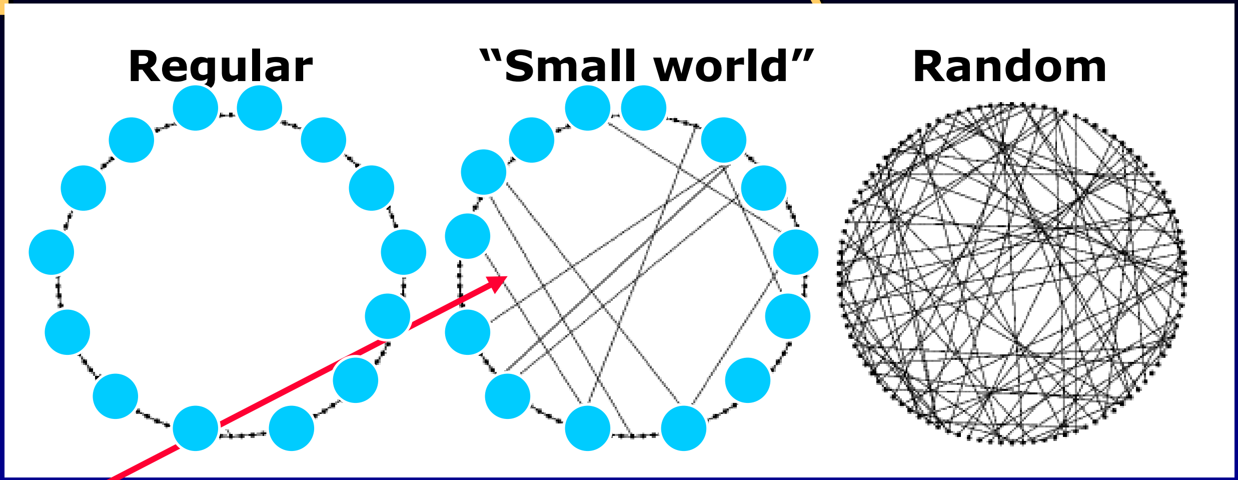

*From Beggs and Plenz (J. Neuroscience, Dec. 2003)*

Complex Networks are the squeletum  of a Complex System

# Networks are made of nodes connected by links

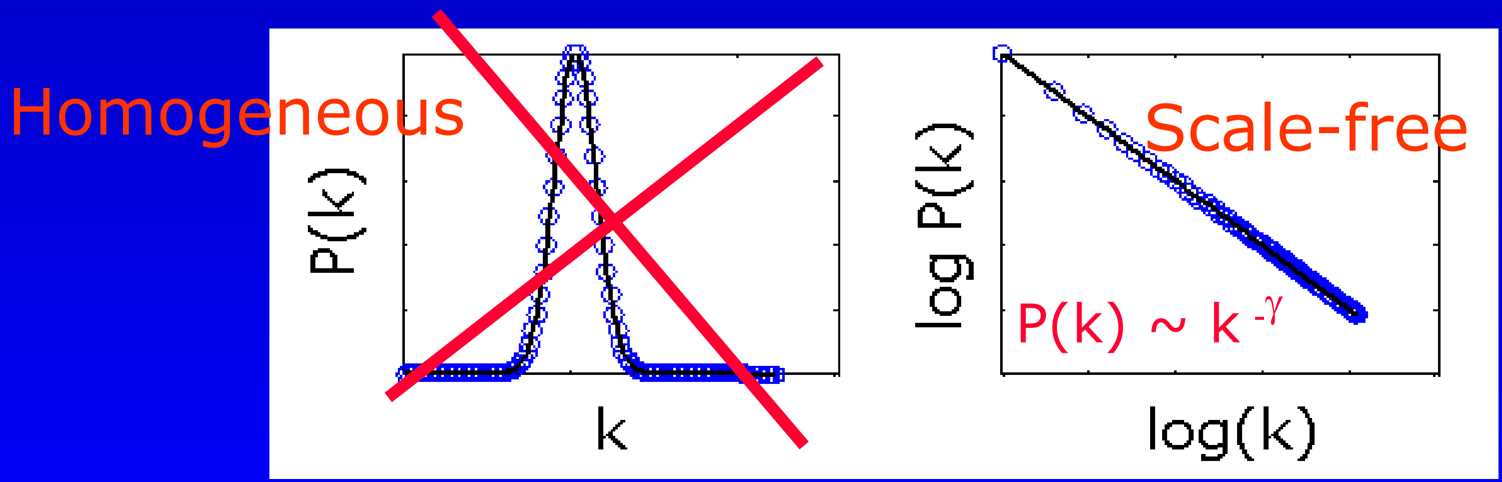


# Complex Networks (in 60 sec.)




A few shortcuts on a regular net make a small world network (which have the clustering of the regular and the short path length of the random net).

## Networks in nature are not homogeneous!




In random nets most nodes are linked by about the same number of links (k), while in scale-free nets a few are extremely well connected.

# Networks Statistical Prop. (in 10 sec.)

- Degree distribution: $P(k) \sim k^{-\gamma}$
  (how many links each node have )
- Average shortest distance: L
  (shortest distance between any two nodes)
- Clustering: $C(k) \sim k^{-\mu}$ (how many of your links are also mutually linked)
- Average connectivy of neighbors: $K_{nn}(k) \sim k^{-\delta}$
  (how many links my neighbor have)

( Rev. Mod. Phys. 74, 47 (2002); Adv. in Physics 51, 4 (2002); SIAM Rev. 45, 167 (2003) )

# Why we care about "scale-free" networks

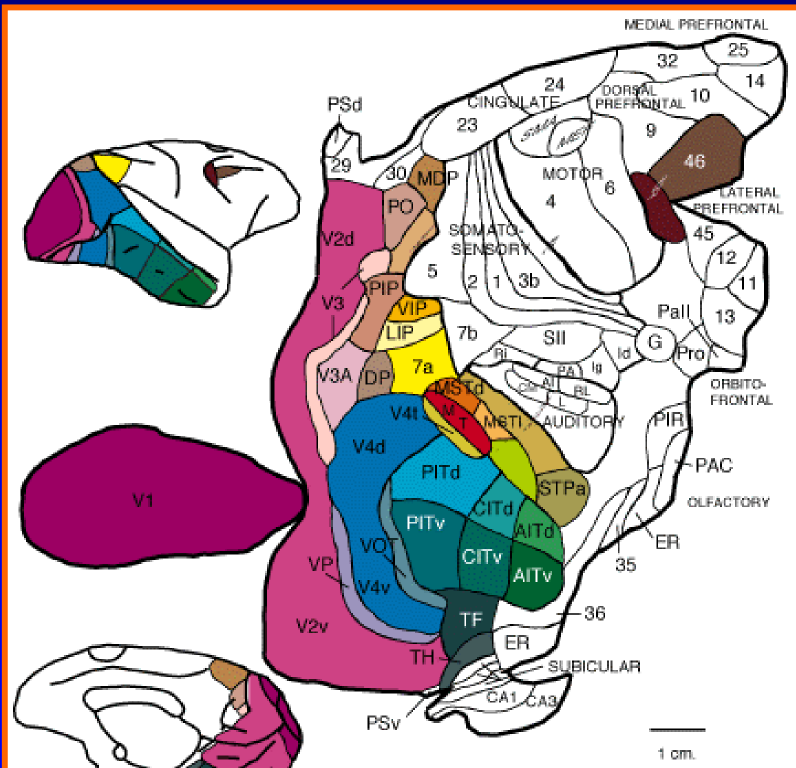
- They are highly clustered and at the same time have short minimal length

(sort of well connected at all scales)

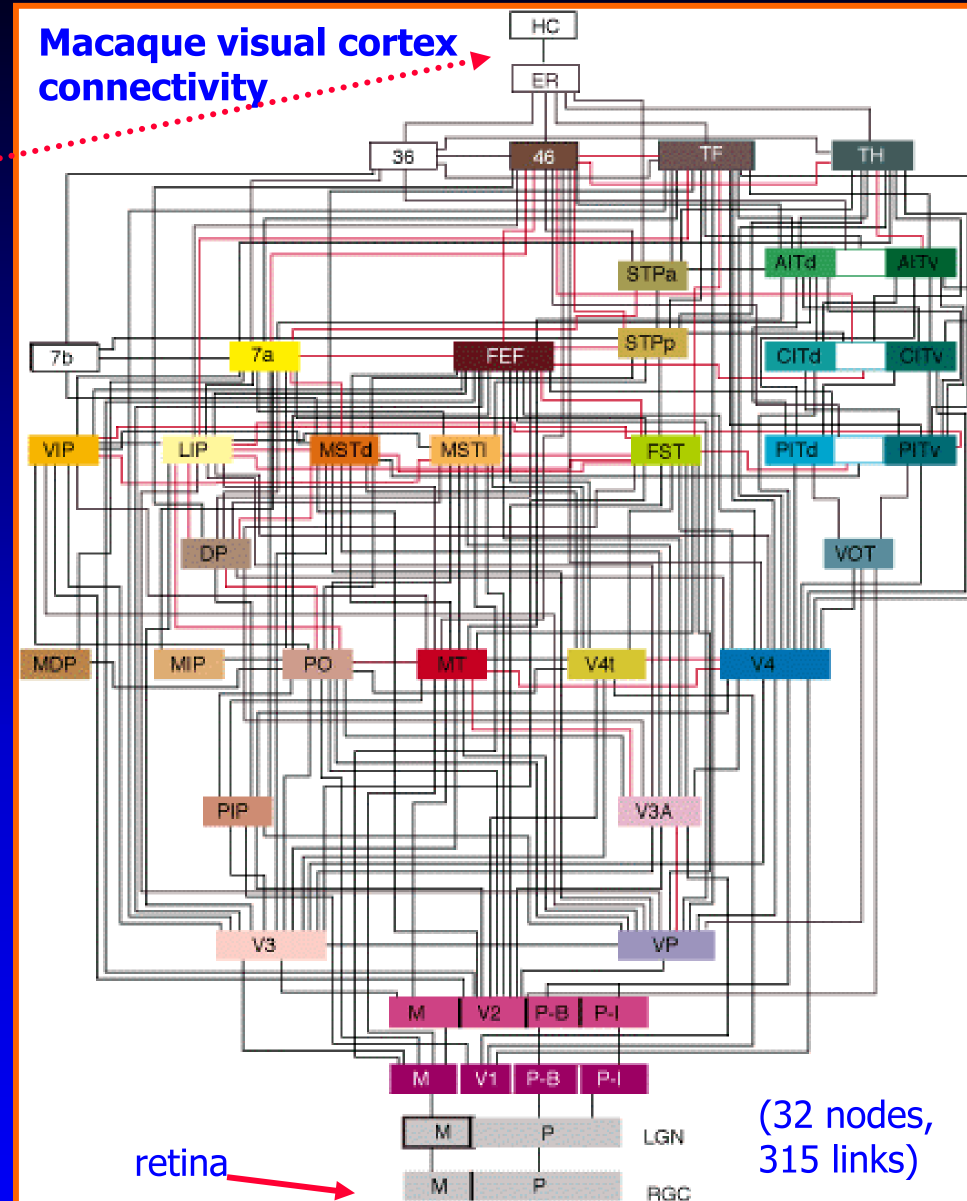
- Faster synchronizability.
- In terms of resistance to damage: are Robust (to random) and Fragile (to targeted attack)

# “In catalogue” cortical nets

Just by visual inspection one suspects that this network can not be scale-free
Note, for example HC (hippocampus) have degree 1



Flat map of macaque cortex

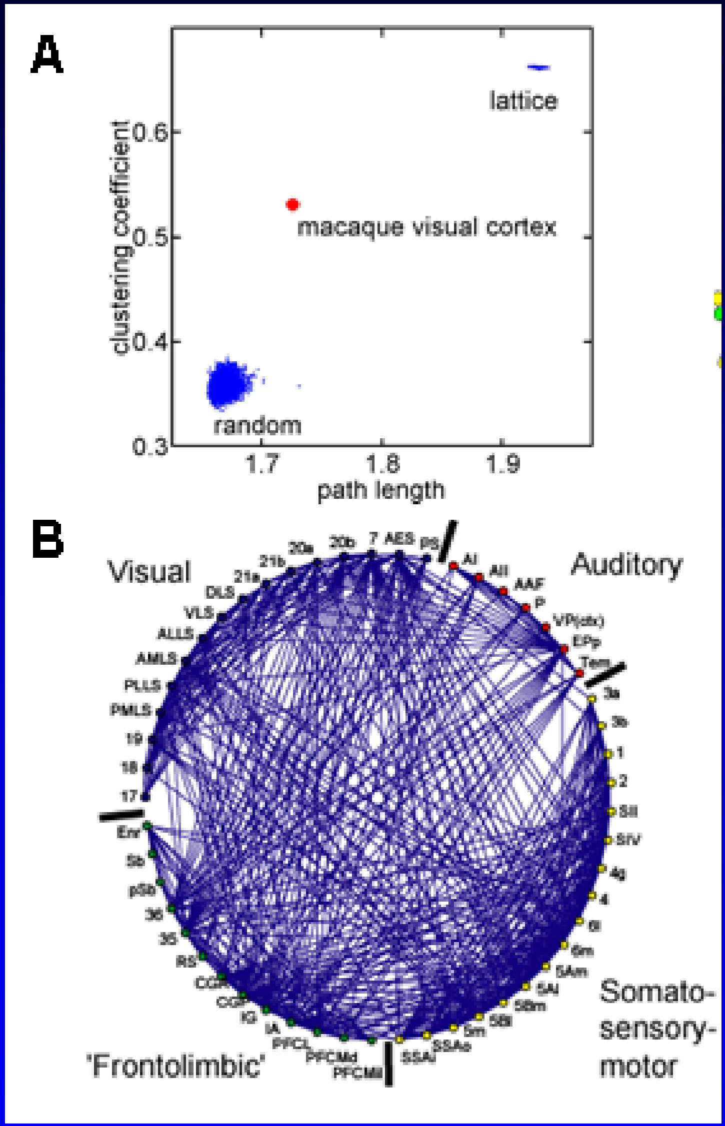


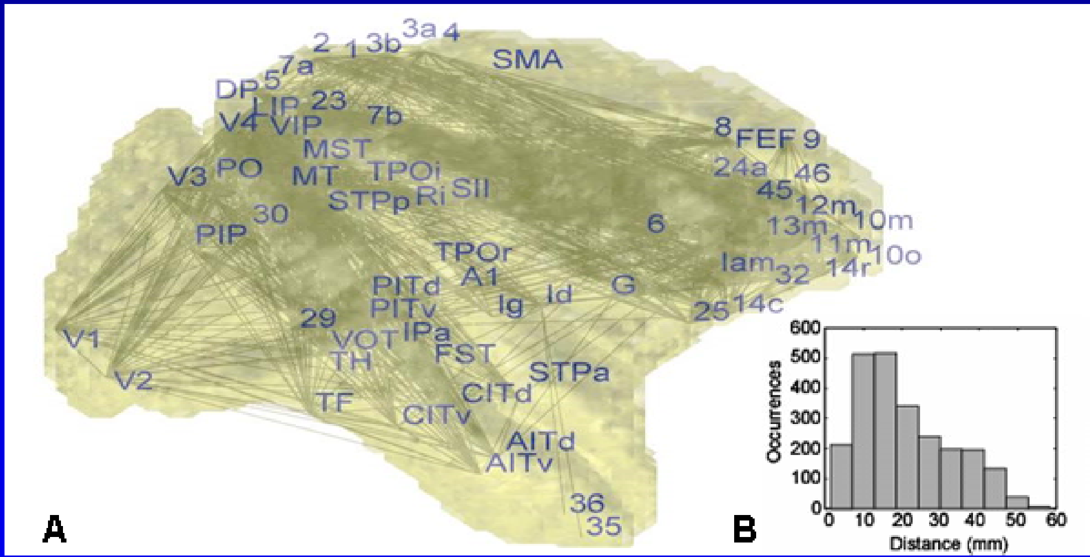
Macaque visual cortex connectivity

(32 nodes, 315 links)

From Felleman and Van Essen, (1991) *Cerebral Cortex* 1:1-47.

# In catalogue cortical nets

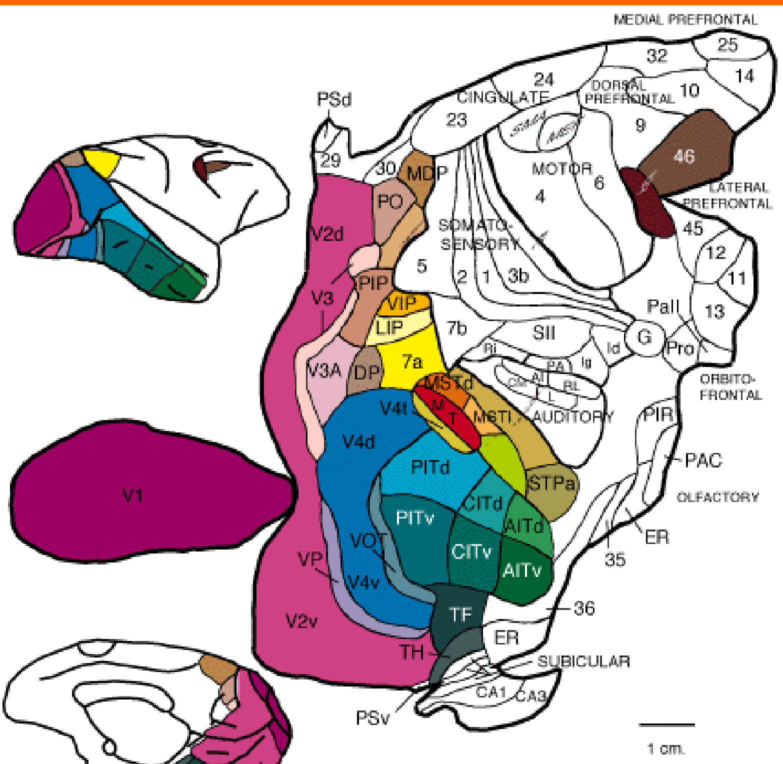


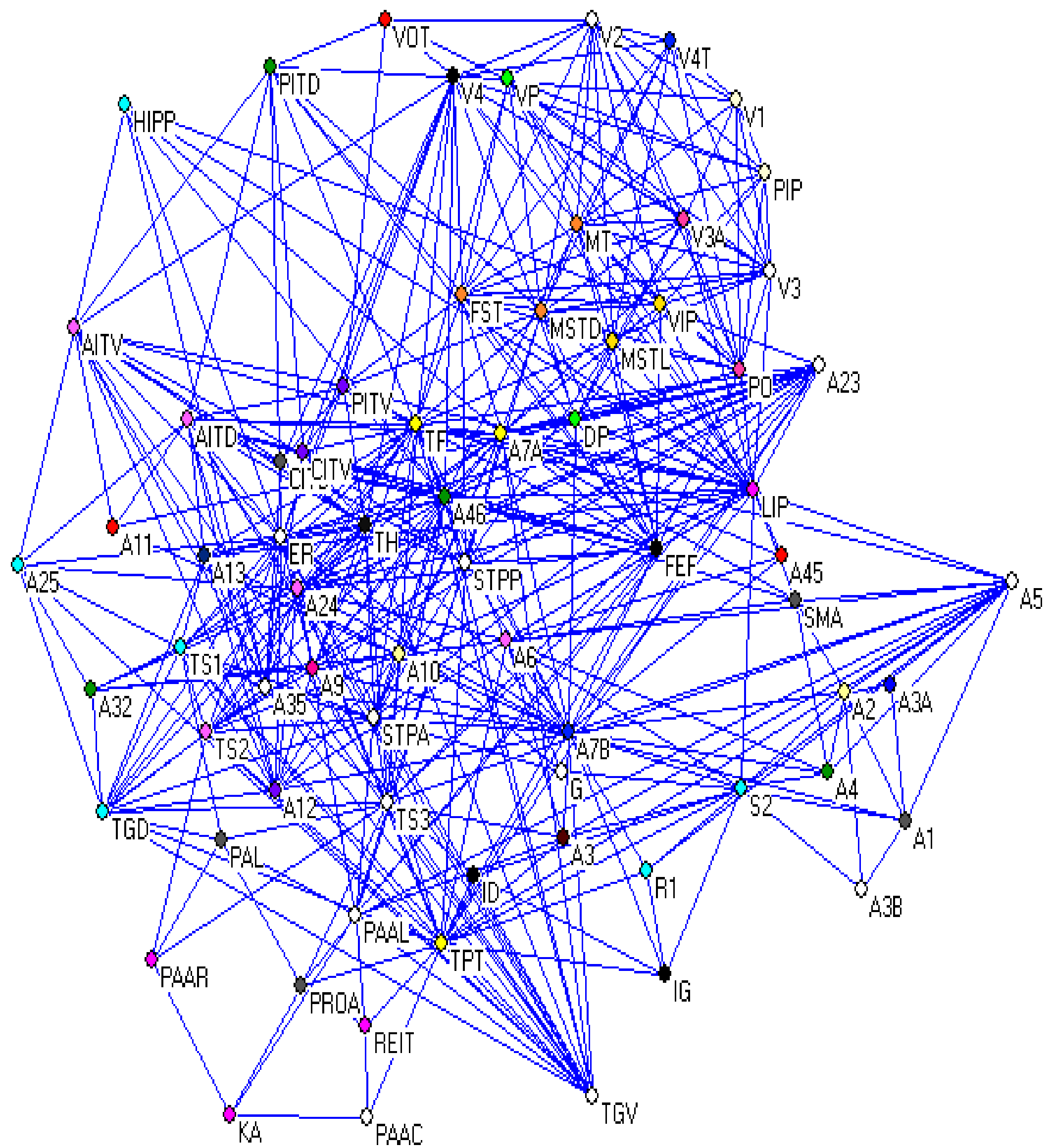

*From: Organization, Development and Function of Complex Brain Networks. Sporns, Chialvo, Kaiser and Hilgetag. Trends in Cognitive Sciences 2004*

# The most complete cortical network in the literature

Entire macaque cerebral cortex
(71 nodes, 755 links)




Flat map of macaque cortex



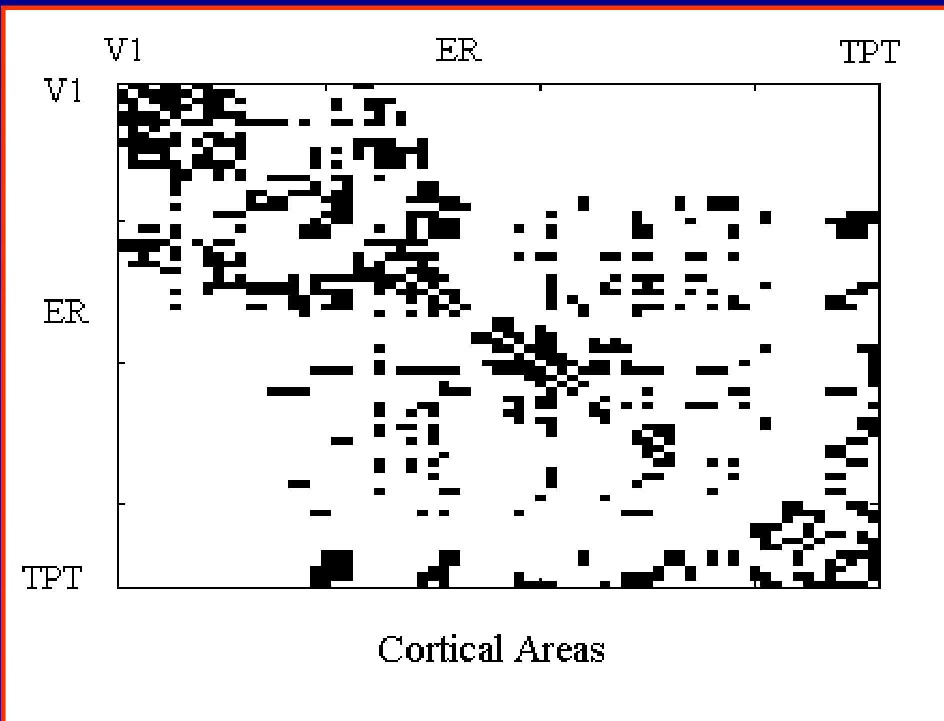

From Sporns et al, *Cerebral Cortex*, 10:127-141(2000).

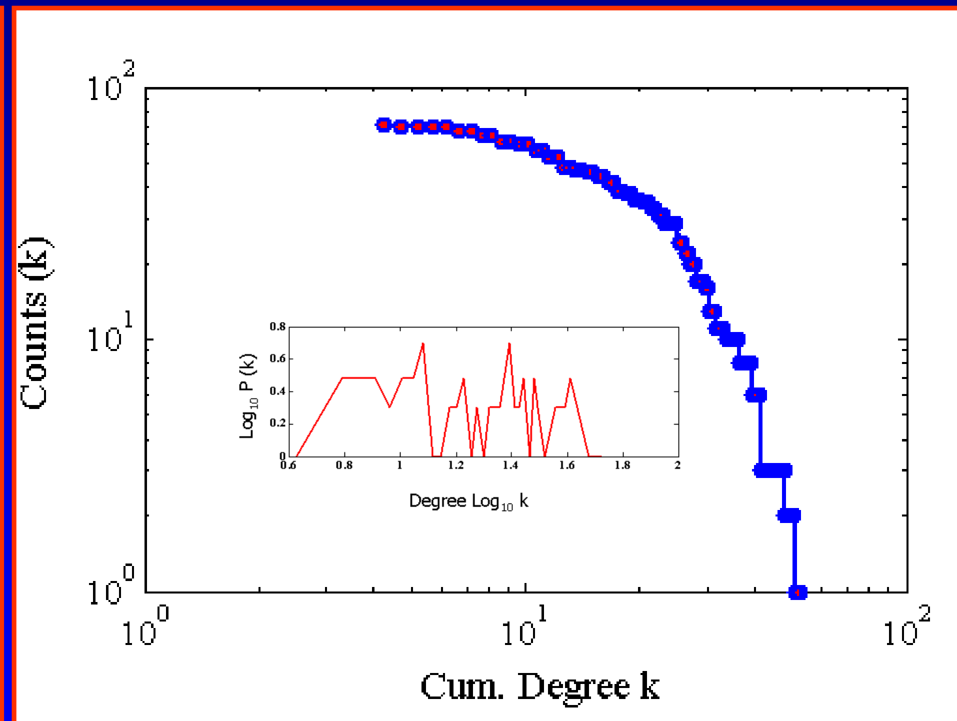
# …"Catalogue" nets are small-world but not scale-free (very homogeneous)

| *Network* | *N* | *C* | *L* | *<k>* | . | $C_{rand}$ | $L_{rand}$ |
|---|---|---|---|---|---|---|---|
| Macaque CC | 71 | 0.46 | 2.3 | 10.6 | . | 0.15 | 2.0 |

"Small-world"

- $C >> C_{rand}$
- $L \sim L_{rand}$

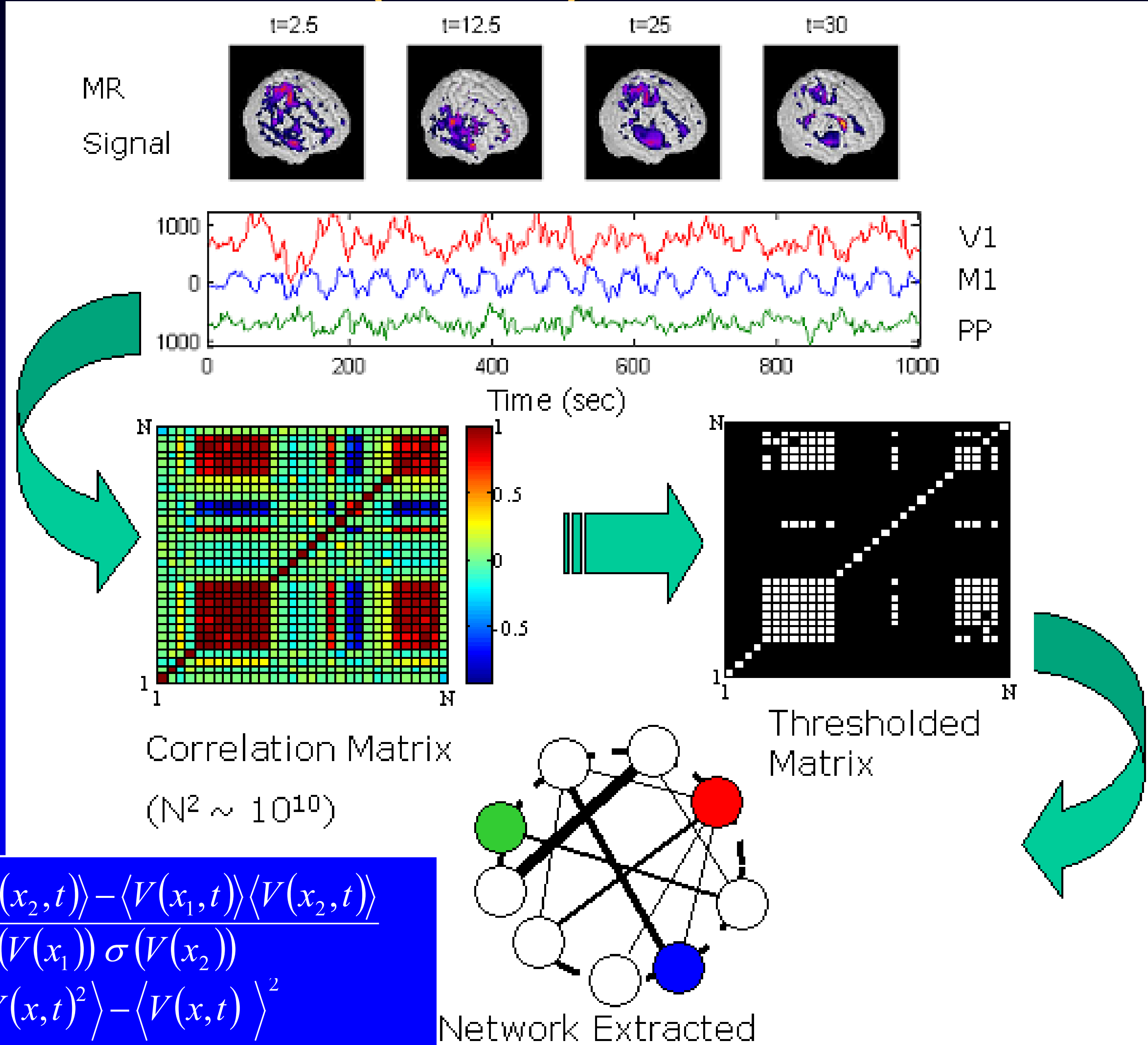



Macaque cerebral cortex

# Functional Magnetic Resonance Imaging ("fMRI")

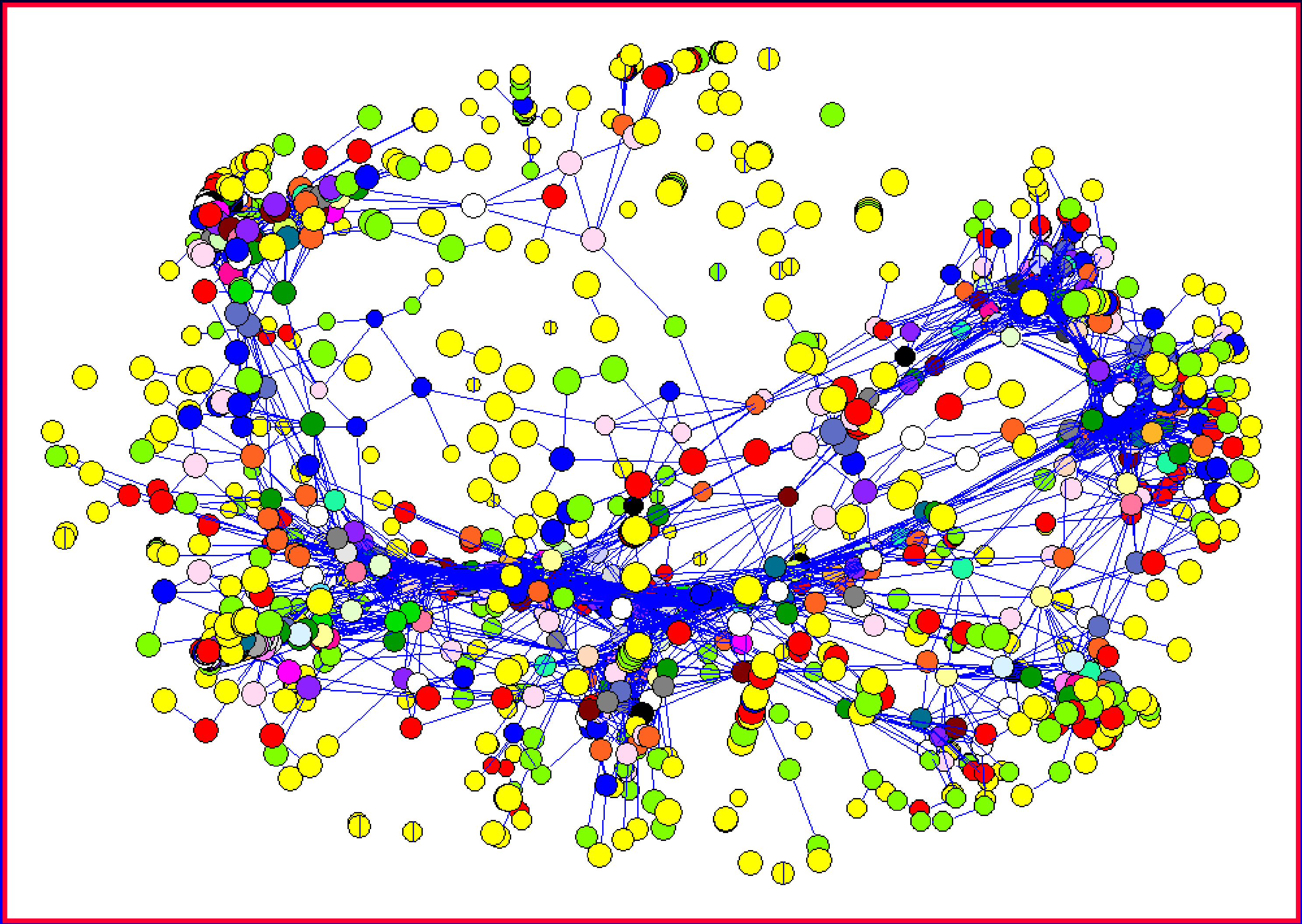
# "In vivo" brain nets (Fmri)

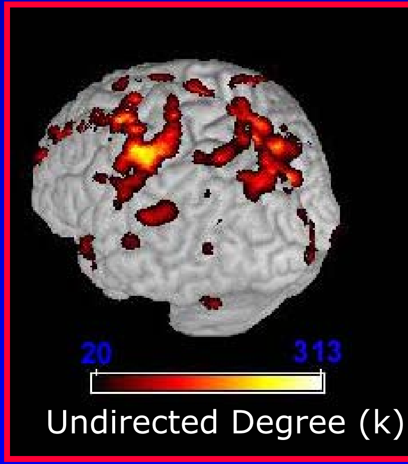



$$r(x_1,x_2)=\frac{\langle V(x_1,t)\quad V(x_2,t)\rangle-\langle V(x_1,t)\rangle\langle V(x_2,t)\rangle}{\sigma(V(x_1))\,\sigma(V(x_2))}$$

$$\sigma^2(V(x))=\langle V(x,t)^2\rangle-\langle V(x,t)\ \rangle^2$$

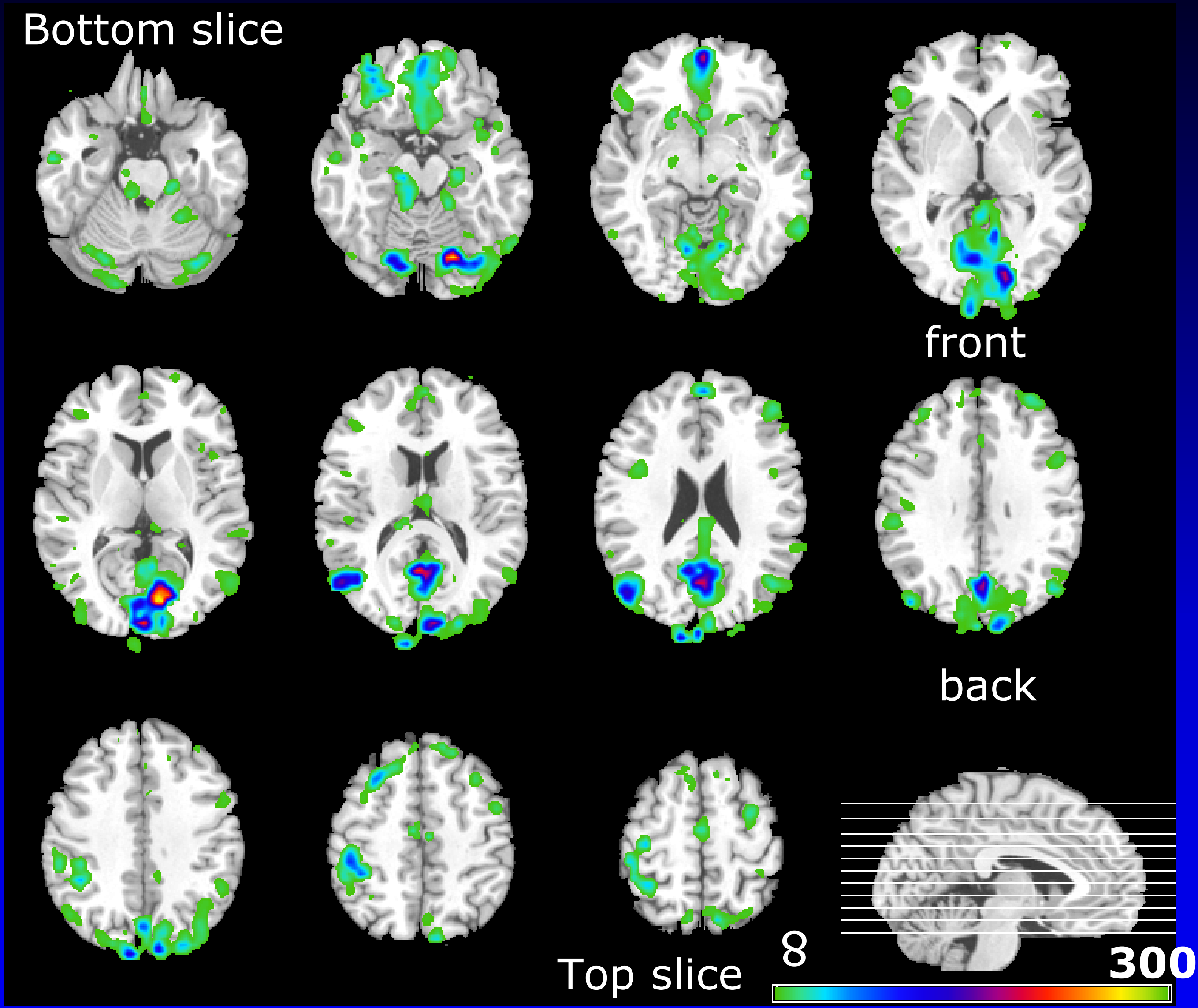
# Brain' Net (finger tapping)






Colors indicate the number of links (degree) of each node. yellow=1, green 2, red=3, blue=4, etc

# Hubs

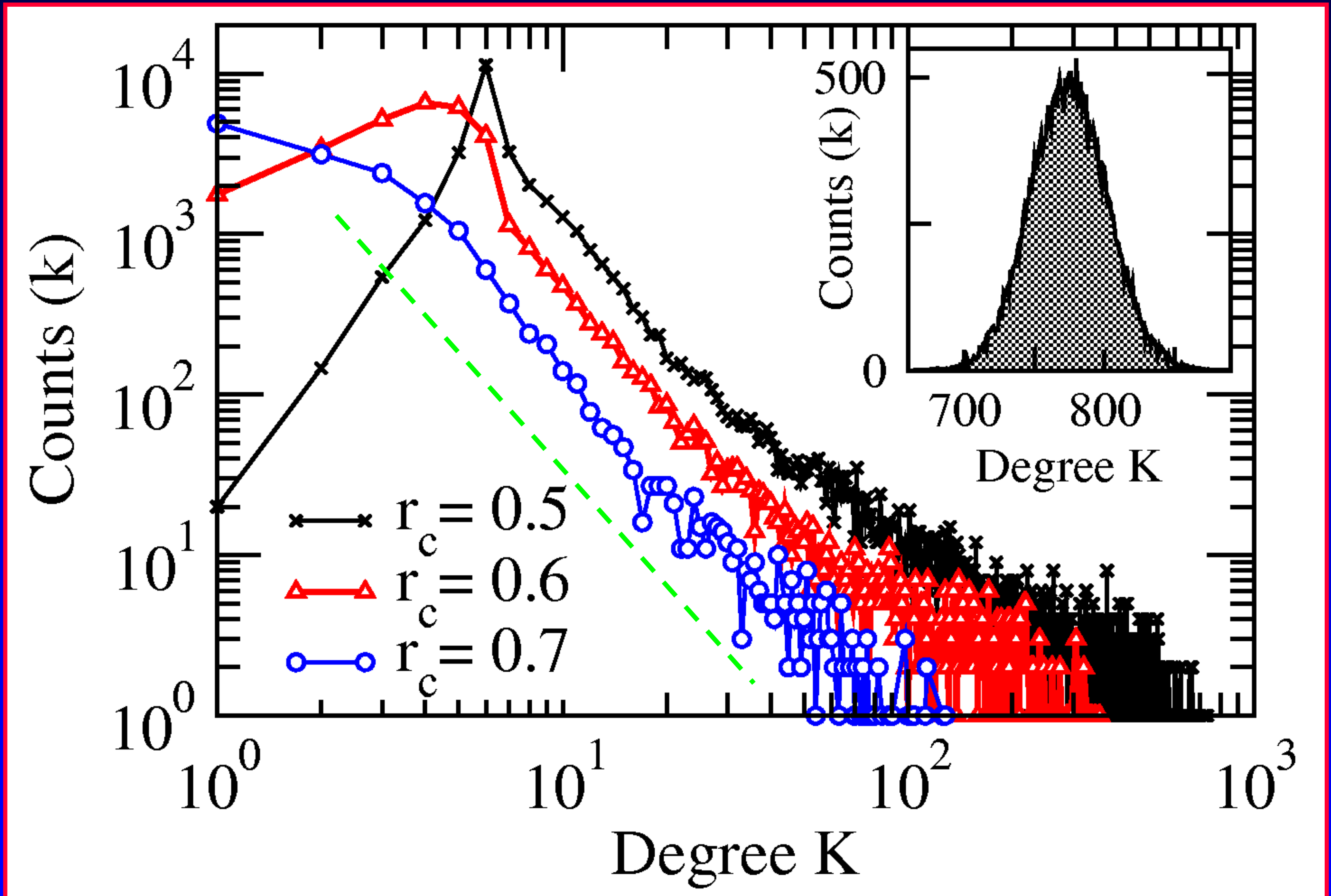



Undirected Degree (k)

Colors indicate number of links (degree) of each site

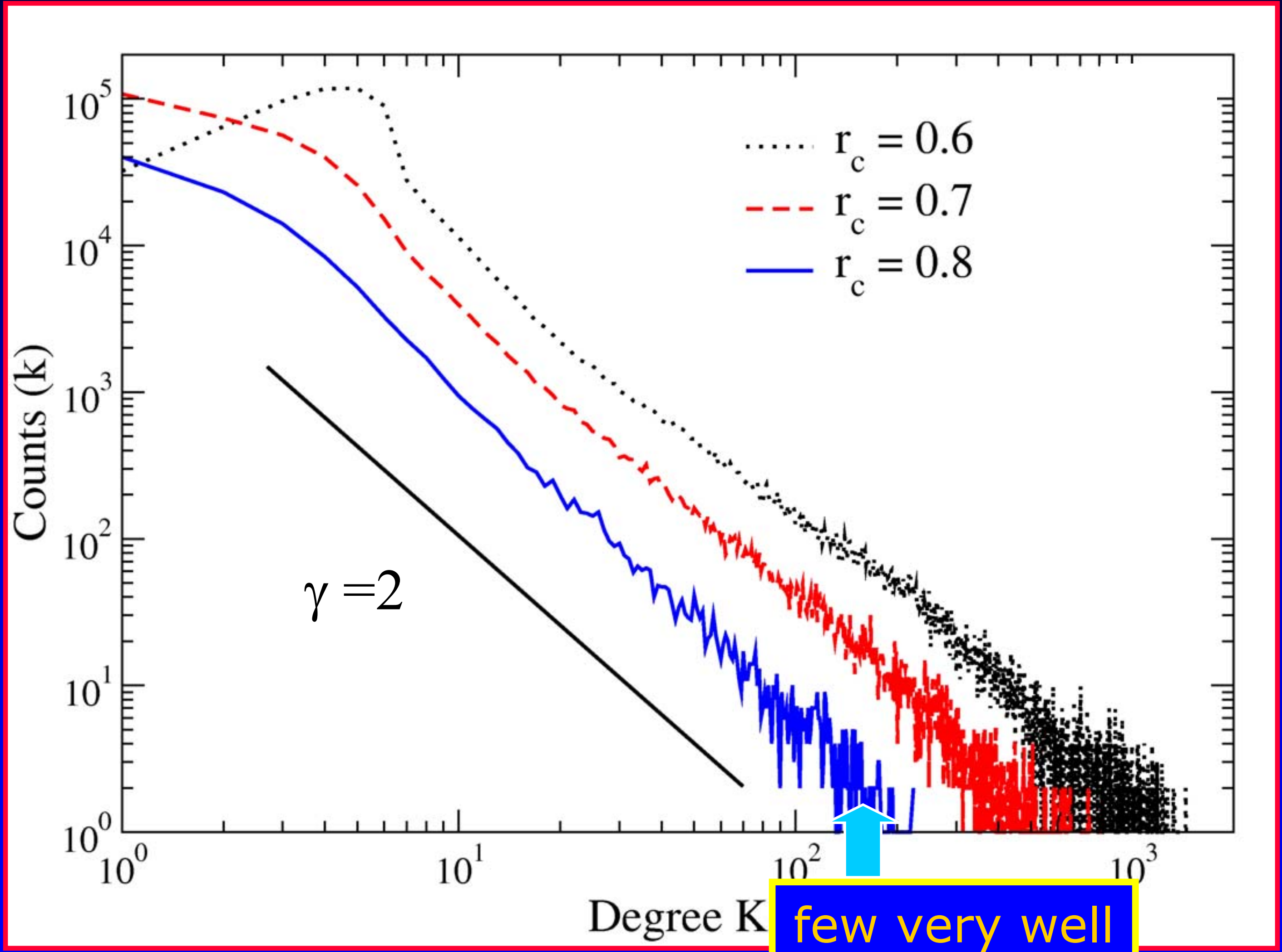
# Degree Distribution (i.e., how many links each node have) of my brain

**Scale-free $k^{-\gamma}$ with $\gamma \sim 2$**



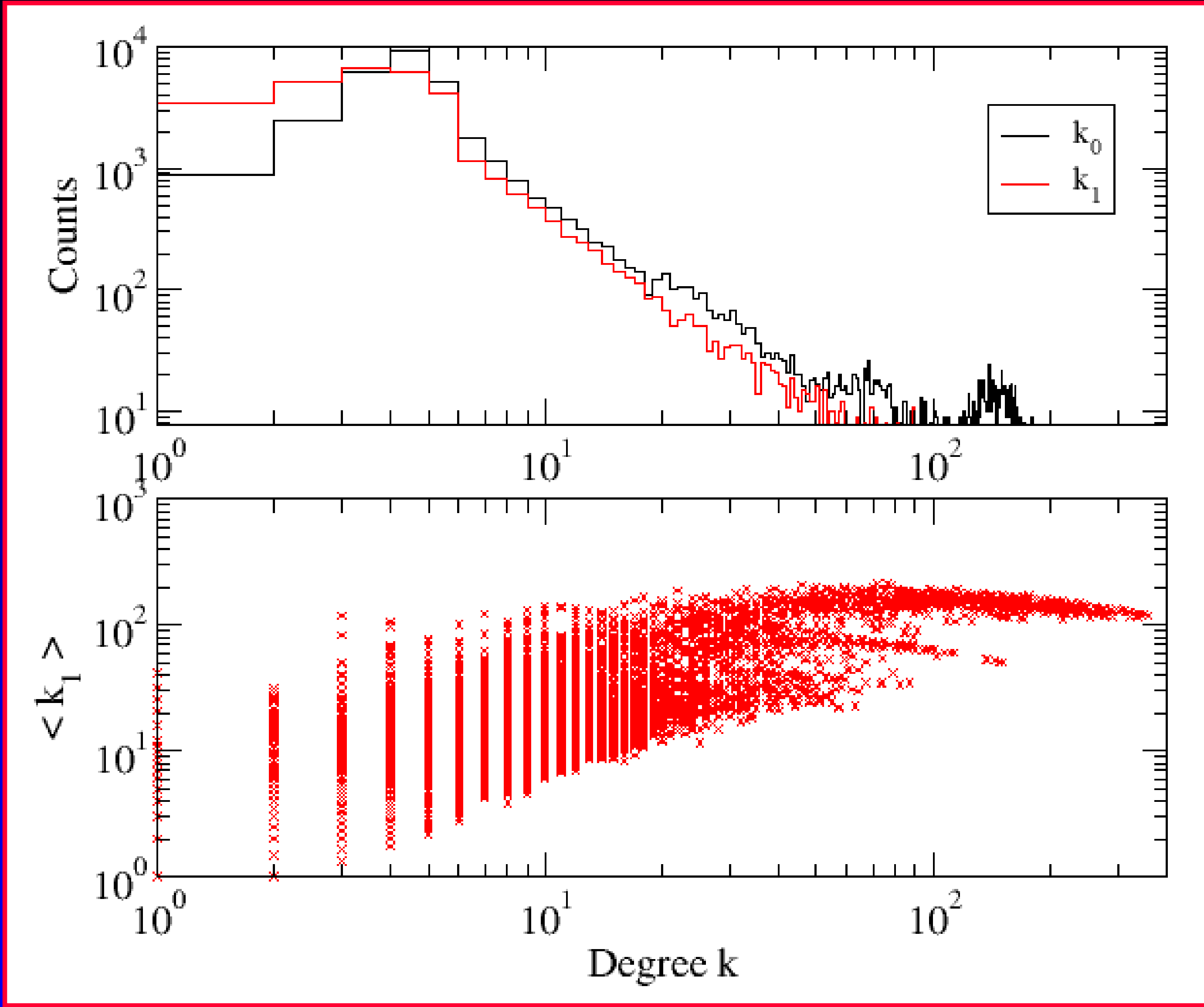
# Average Degree Distribution

n=22 from 7 subjects




few very well connected brain sites

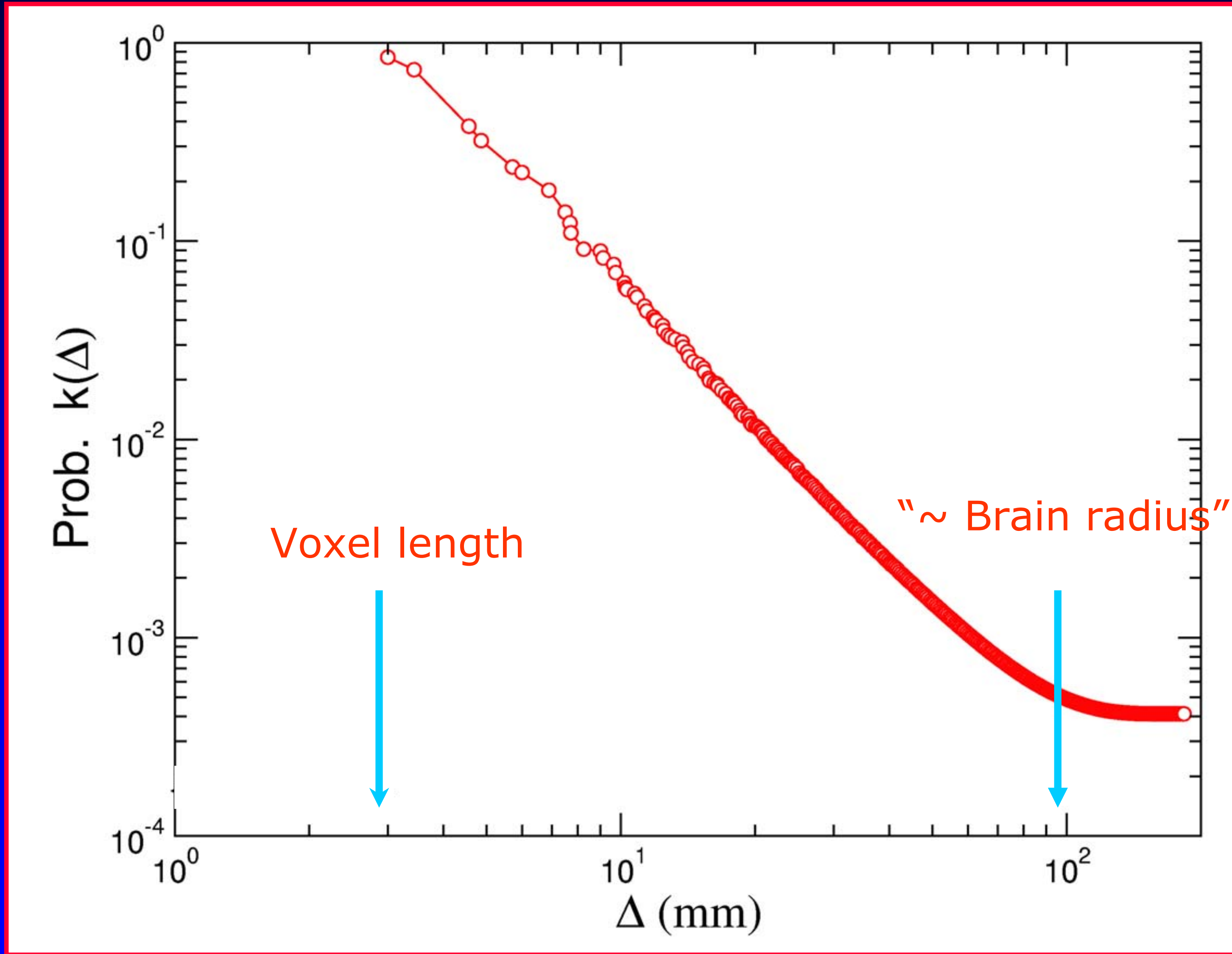
# K0-K1



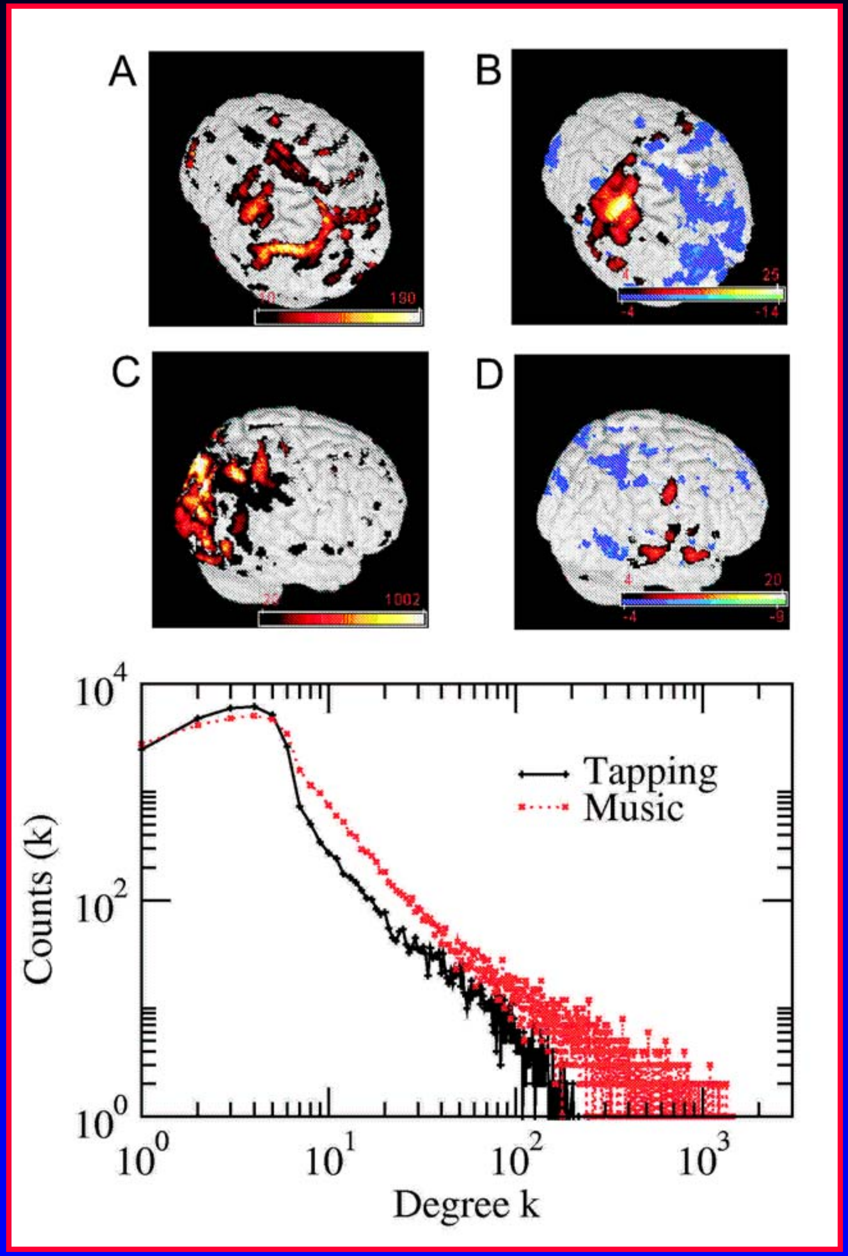

- Neighbors of well connected nodes are also well connected (“Assortative”)

# Average Links Length Distribution

Probability of finding a link between two nodes separated by a distance **x < Δ**



Different tasks
Different networks
Similar scaling



# Statistics

## fMRI-results

| $r_c$ | $N$ | $C$ | $L$ | <k> | $\gamma$ | $C_{rand}$ | $L_{rand}$ |
|---|---|---|---|---|---|---|---|
| 0.6 | 31503 | 0.14 | 11.4 | 13.41 | 2.0 | $4.3 \times 10^{-4}$ | 3.9 |
| 0.7 | 17174 | 0.13 | 12.9 | 6.29 | 2.1 | $3.7 \times 10^{-4}$ | 5.3 |
| 0.8 | 4891 | 0.15 | 6. | 4.12 | 2.2 | $8.9 \times 10^{-4}$ | 6.0 |

## Previous related results

| *Network* | $N$ | $C$ | $L$ | <k> | . | $C_{rand}$ | $L_{rand}$ |
|---|---|---|---|---|---|---|---|
| C. Elegans[1] | 282 | 0.28 | 2.65 | 7.68 | . | 0.025 | 2.1 |
| Macaque VC[2] | 32 | 0.55 | 1.77 | 9.85 | . | 0.318 | 1.5 |
| Cat Cortex[2] | 65 | 0.54 | 1.87 | 17.48 | . | 0.273 | 1.4 |

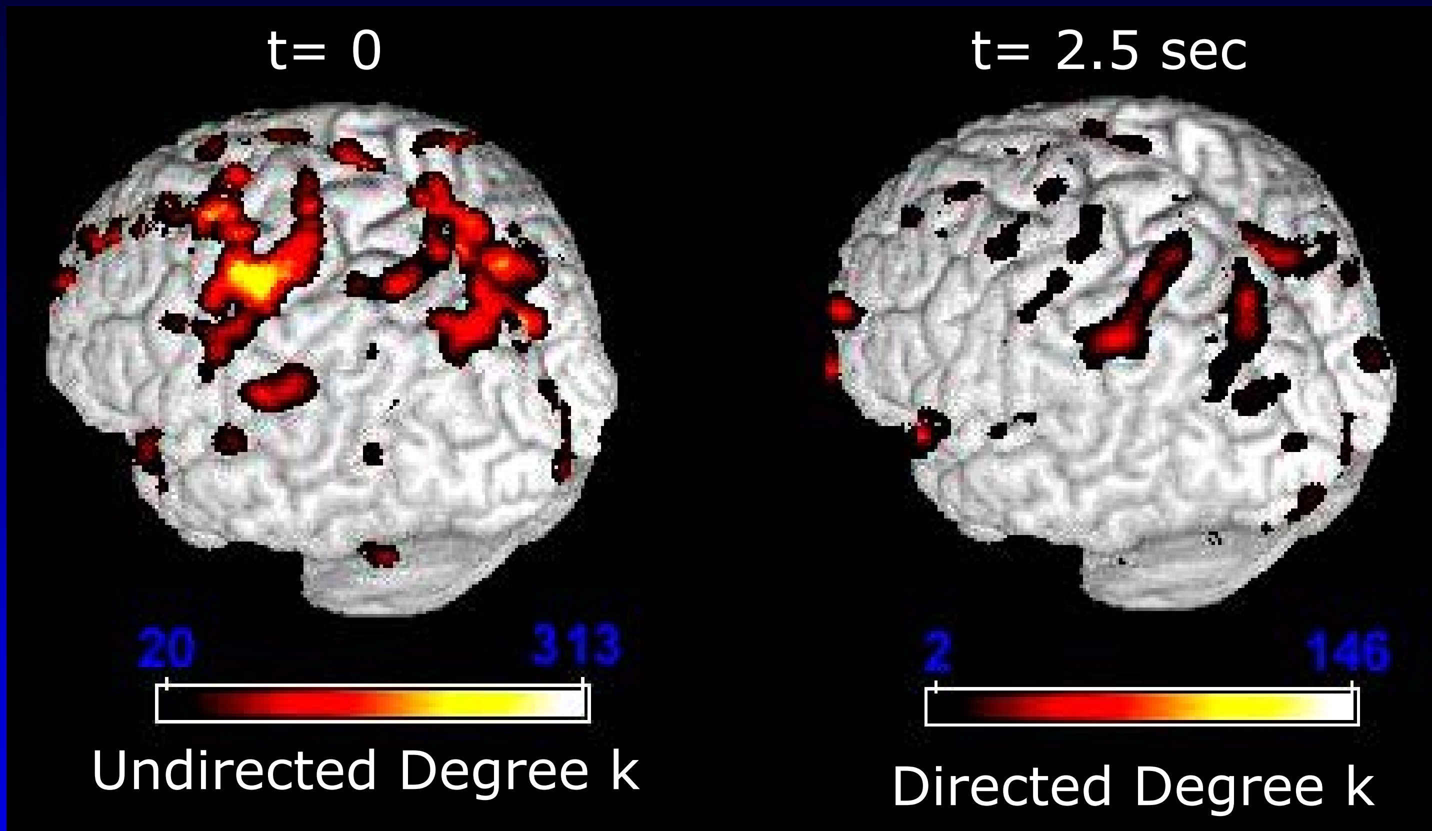
"Small-world"

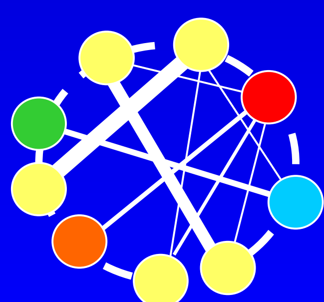
- $C >> C_{rand}$
- $L \sim L_{rand}$

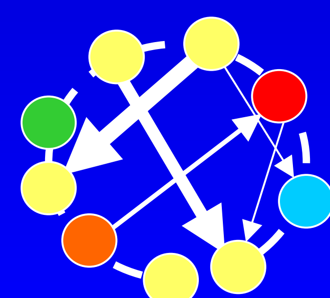
* $C_{rand} \sim$ **<k>/N**

(1) Watts & Strogats, 1998.
(2) Osporn et al, 2003.

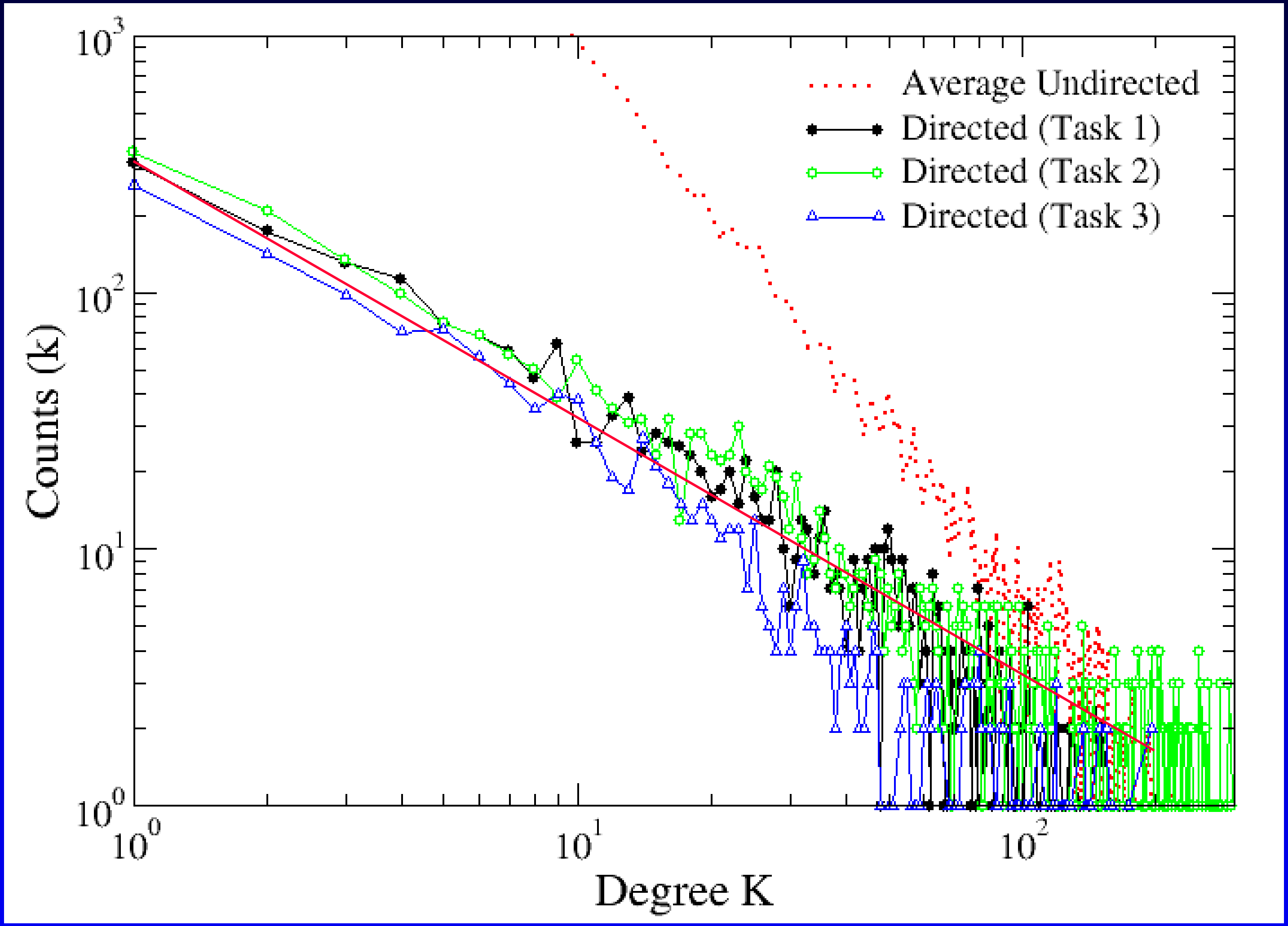
# Hubs II

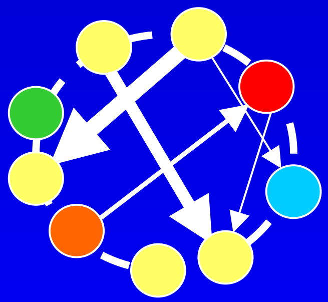




# Directed degrees distr.



Finale

# Blah-blah-logy:

- “In vivo” brain activity do not have a characteristic scale (“scale-free” networks). Some physicists will be happy to know that, after all, the brain is a scale-free network with small-world properties. ($C >> C_{rand}$ ).
- Some biologists will be unhappy to know that “In catalogue” nets are homogeneous (underreported findings?).
- Assortative features …?
- The scale-free character emphasizes the need to talk in terms of effective, functional and structural connectivity. “In numero” networks with similar properties could be hiding surprises.
- The fMRI method allows, in principle, to study the brain in a dance rather than in a pose and address dynamical states as emotion, pain, pleasure, etc).

# Brains are critical

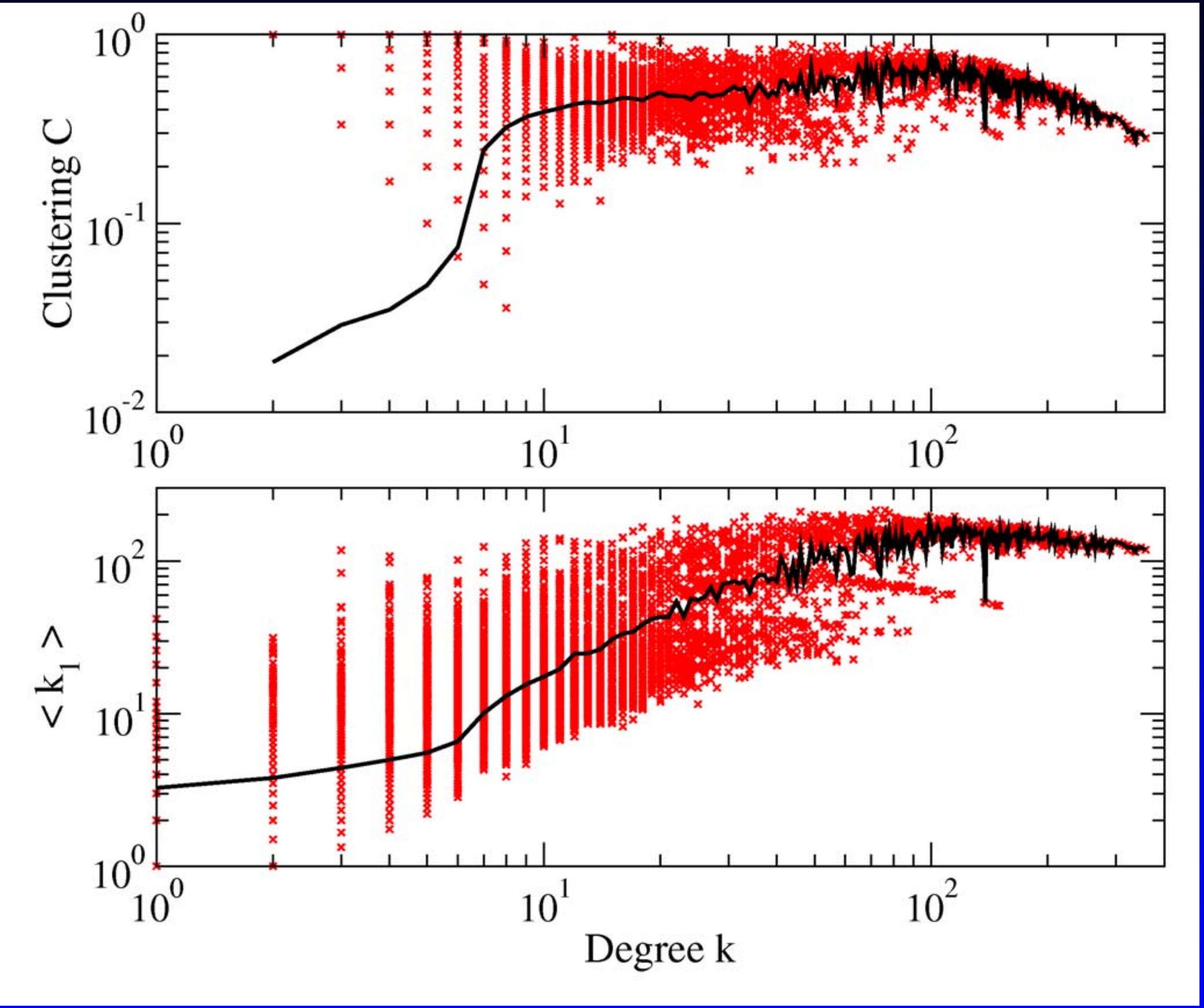
***"Per, para mi el cerebro es critico"...***

***"si, para mi tambien Dante!"***

Per Bak (1947-2002)

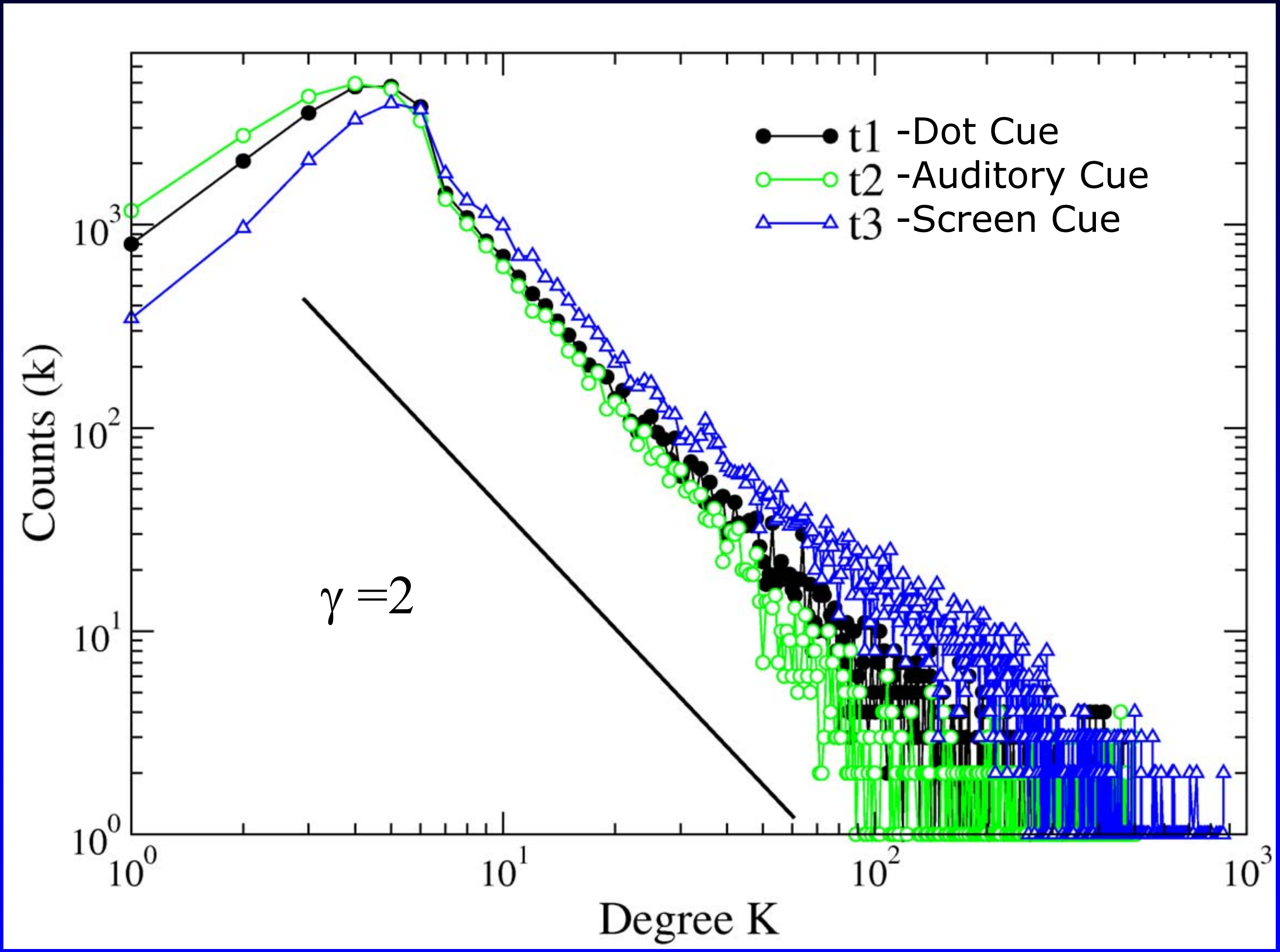
"How Nature Works"
Oxford University Press.

# Degree vs clustering



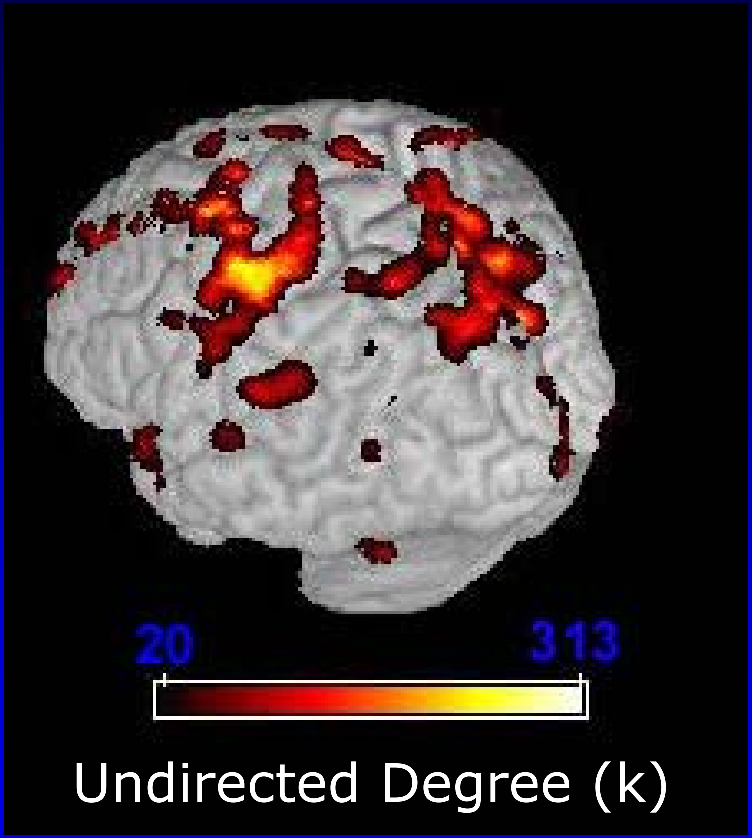

- Clustering is rel. independent of connectivity.

# Another subject in different tasks:

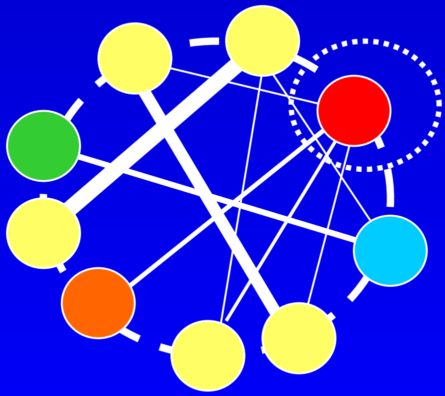



Similar tail decay in different finger tapping tasks

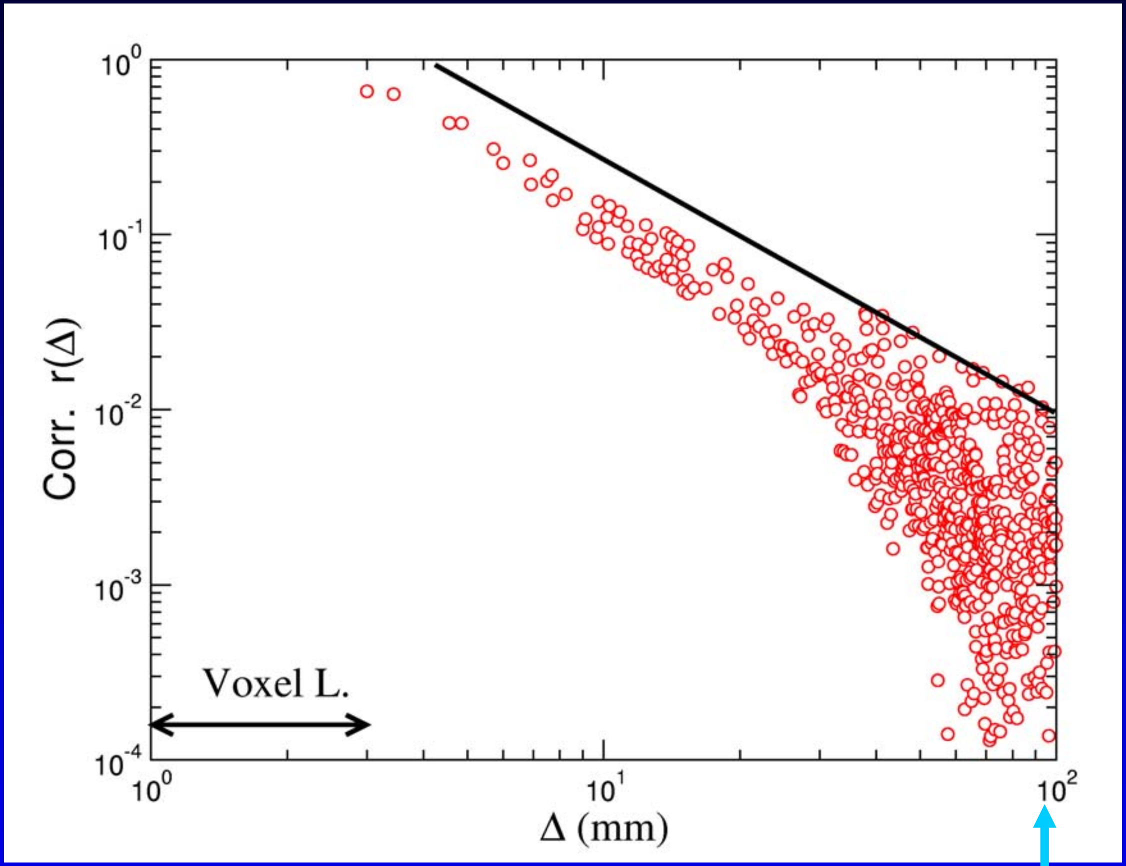
# Hubs




Colors indicate the number of links (degree) of each site.



# Brain "Two-point Correlation"



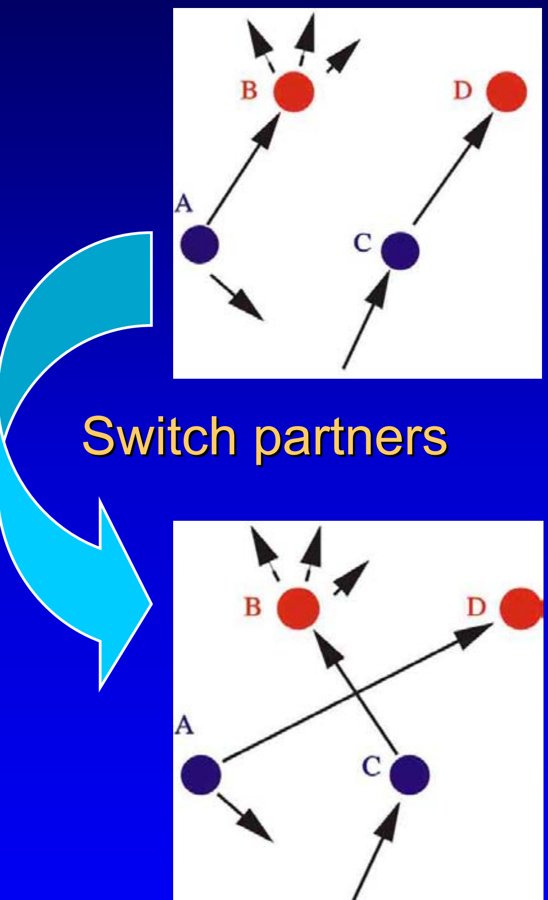

"Brain radius"

$$r(x_1,x_2)=\frac{\langle V(x_1,t)\quad V(x_2,t)\rangle-\langle V(x_1,t)\rangle\langle V(x_2,t)\rangle}{\sigma(V(x_1))\,\sigma(V(x_2))}$$

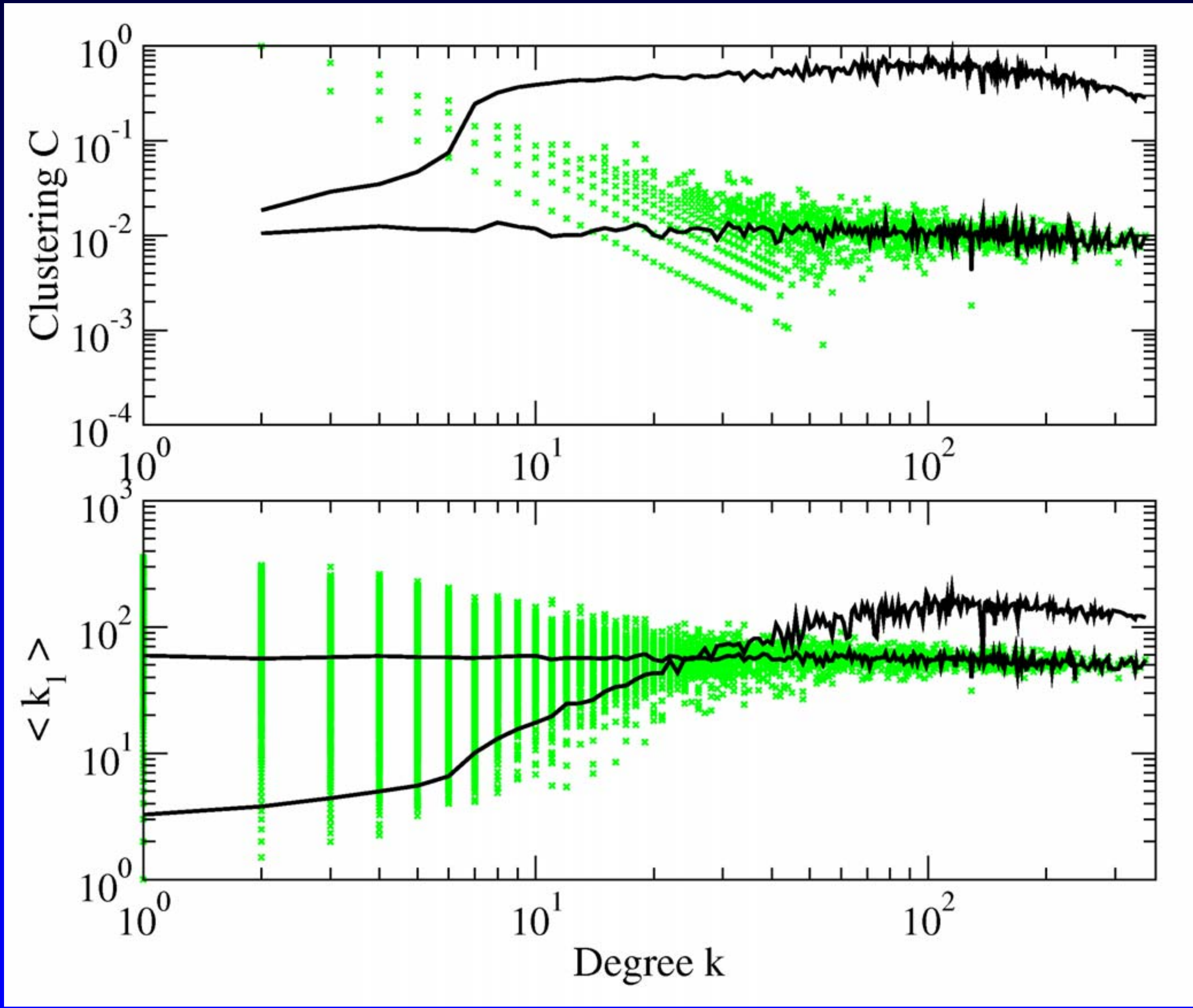
$$\sigma^2(V(x))=\langle V(x,t)^2\rangle-\langle V(x,t)\rangle^2$$

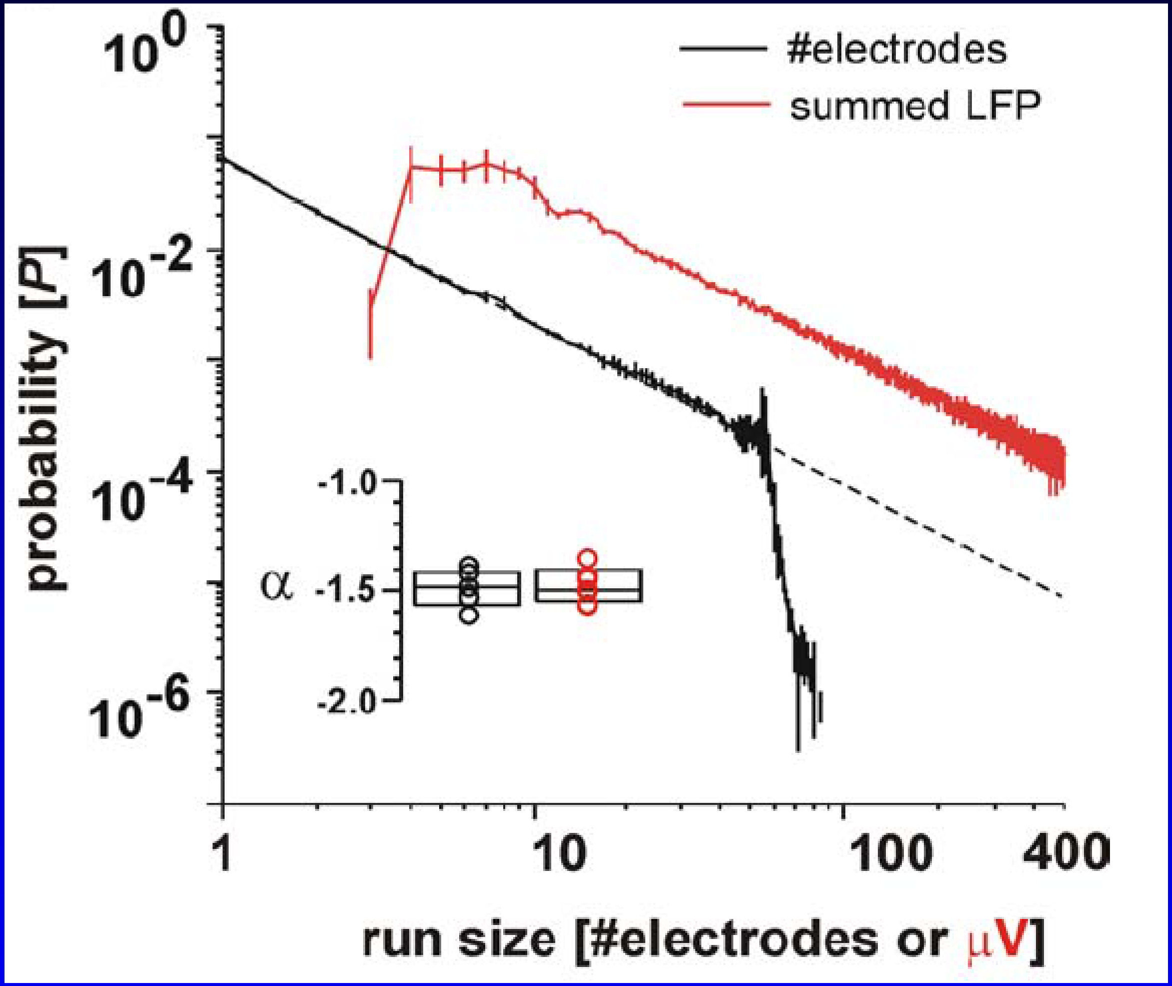
# $K_0$-$K_1$, and Degree vs Clustering

Maslov' rewiring




Switch partners






Begs and Plenz (J. Neuroscience, Dec. 2003)